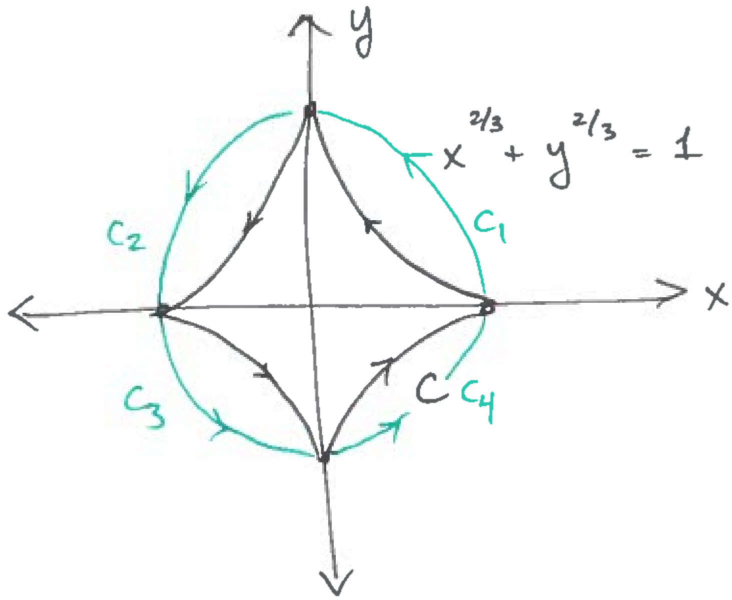

C is a closed curve, but the region enclosed by C contains point not in the domain of

$$F = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle.$$

(i.e. the origin)

$\Longrightarrow$ No Greens theorem

But it was shown that F is a conservative vector field on its domain (c.f. LQ29)

So we can use <u>independence of paths</u> !

$C_1 : \langle \cos t, \sin t \rangle \quad (0 \le t \le \frac{\pi}{2})$

$C_2 : \langle \cos t, \sin t \rangle \quad (\frac{\pi}{2} \le t \le \pi)$

$C_3 : \langle \cos t, \sin t \rangle \quad (\pi \le t \le \frac{3\pi}{2})$

$C_4 : \langle \cos t, \sin t \rangle \quad (\frac{3\pi}{2} \le t \le 2\pi)$

$$\int_C F\,dr = \int_{C_1} F\,dr + \int_{C_2} F\,dr + \int_{C_3} F\,dr + \int_{C_4} F\,dr$$

Notice that $F(r(t)) \cdot r'(t) = \left\langle -\frac{\sin t}{1}, \frac{\cos t}{1} \right\rangle \cdot \langle -\sin t, \cos t \rangle$

$$= \sin^2 t + \cos^2 t = 1.$$

Hence $\int_{C_1} F\,dr = \int_0^{\frac{\pi}{2}} 1\,dt = \frac{\pi}{2}$, and likewise $\int_{C_i} F\,dr = \frac{\pi}{2}$ for each $1 \le i \le 4$.

So then $\oint_C F\,dr = \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} = \boxed{2\pi.}$