Quiz 1
Due: 3 September 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: ______________________________

1. **Justify** whether the following statement is True or False: $\frac{23\pi}{6}$ radians $>$ $400°$.

Solution: Let's convert $\frac{23\pi}{6}$ radians into degrees so that we can compare the two. We compute this as

$$\frac{23\pi}{6} \text{ radians} = \frac{23\pi}{6} \text{ radians} \cdot \left(\frac{180°}{\pi \text{ radians}}\right) = \frac{23 \cdot 180°}{6} = 690°.$$

Hence $\frac{23\pi}{6}$ radians $= 690°$ which is greater than $400°$, so the statement is $\boxed{\text{TRUE.}}$
(These numbers are too large to appear on an exam without calculators.)

2. Determine the angle θ between 0 and 2π that is coterminal to 70801π.

Solution: Since 70801 is an odd integer, when we repeatedly subtract 2π from 70801π we will eventually reach $\ldots, 7\pi, 5\pi, 3\pi, \ldots$ and then π. In other words, we have $70801\pi = 70800\pi + \pi$ and

$$70800\pi = 35400 \cdot 2\pi = \underbrace{2\pi + 2\pi + \ldots + 2\pi}_{35400 \text{ times}}.$$

So 70800π is coterminal to 0 and thus 70801π is coterminal to π. Hence $\boxed{\theta = \pi.}$

3. The London Eye is a huge Ferris wheel and the most popular paid tourist attraction in the United Kingdom. Riders board from a platform that is 2 meters above the ground, and at the highest point on the ride they are 137 meters above the ground. It completes one rotation every 30 minutes.
   What is the **linear speed** of the London Eye in meters per hour?

> **Solution:** The wheel makes 2 revolutions every **hour**. Hence the angular speed of the London Eye is
>
> $$\omega = \frac{\theta}{t} = \frac{2 \cdot 2\pi \text{ radians}}{1 \text{ hour}} = \frac{4\pi \text{ radians}}{1 \text{ hour}}.$$
>
> The linear speed is given by $v = r\omega$, where r is the radius of the wheel. At the bottom the riders are 2 meters off the ground and at the top they are 137 meters off the ground, so the diameter of the wheel is $137 - 2 = 135$ meters. Hence the radius is $r = 135/2 = 67.5$ meters. Therefore, the linear speed of the London Eye in meters per hour is
>
> $$v = r\omega = (67.5 \text{ meters}) \cdot \left(\frac{4\pi}{1 \text{ hour}}\right) = \boxed{\frac{270 \text{ meters}}{1 \text{ hour}}}.$$
>
> **(Note that the actual London Eye has a diameter of 120 meters. The time to make one rotation is accurate. You can resolve the problem to find the actual speed is close to 0.6 miles per hour.)**

4. Imagine that you and a friend are at your favorite pizza place.

   a.) The crust on your friend's pizza has an arclength of $\frac{3\pi}{2}$ inches and the angle at the tip is 30°. What is the diameter of the pizza that they ordered a slice of?

   > **Solution:** Arclength is given by $s = r\theta$ with θ measured in radians. So here we have $\frac{3\pi}{2} = r \cdot \frac{\pi}{6}$, where 30° has been converted to $\pi/6$ radians. Solve for r:
   >
   > $$r = \frac{3\pi}{2} \cdot \frac{6}{\pi} = 9.$$
   >
   > Hence the diameter of the pizza is $2r = 2 \cdot 9 = \boxed{\text{18 inches.}}$

b.) You order a personal pizza with a 10 inch diameter. There is one inch of crust around the entire outside of the pizza, and the rest is cheese. You want to make sure that you eat exactly 4π square inches of cheese. To what angle should you cut the tip of your pizza slice to eat?

Solution: The area of a sector is given by $A = \frac{1}{2}\theta r^2$, where θ is measured in radians. We want to cut a slice of the pizza described above so that the sector area of cheese is exactly 4π. The radius of this sector is $5 - 1 = 4$ inches because the last inch is just crust. Thus $4\pi = \frac{1}{2}\theta(4)^2 = 8\theta$. Solve for θ:

$$\theta = \frac{4\pi}{8} = \boxed{\frac{\pi}{2} = 90^\circ.}$$

Hence we can eat **one quarter** of the pizza.