

Quiz 1
21 May 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: _____

1. How do you know that $\vec{u} = \langle -2, 2, 4 \rangle$ and $\vec{v} = \langle 10, -10, -20 \rangle$ are parallel?

Solution: Since $\boxed{-5\vec{u} = \vec{v}}$ we know that \vec{u} and \vec{v} are parallel.

2. Give a **unit** vector orthogonal to both $\vec{u} = \langle 1, 2, 3 \rangle$ and $\vec{v} = \langle -3, 2, 4 \rangle$.

Solution: The cross-product $\vec{u} \times \vec{v}$ will be orthogonal to both \vec{u} and \vec{v} . Compute

$$\vec{u} \times \vec{v} = \det \begin{bmatrix} i & j & k \\ 1 & 2 & 3 \\ -3 & 2 & 4 \end{bmatrix} = \langle 2, -13, 8 \rangle.$$

To make this a unit vector we scale by $|\vec{u} \times \vec{v}| = \sqrt{2^2 + (-13)^2 + 8^2} = \sqrt{237}$ to get

$$\pm \left\langle \frac{2}{\sqrt{237}}, \frac{-13}{\sqrt{237}}, \frac{8}{\sqrt{237}} \right\rangle,$$

where the \pm indicates we can choose either direction.

3. What is the center and radius of the sphere given by $x^2 + y^2 + z^2 = -2x + 4y + 4$.

Solution: Rearrange the equation to

$$x^2 + 2x + y^2 - 4y + z^2 = 4.$$

Now complete the squares to get

$$(x + 1)^2 - 1 + (y - 2)^2 - 4 + z^2 = 4$$

from which we see the $\boxed{\text{center is } (-1, 2, 0)}$ and $\boxed{\text{radius is } \sqrt{9} = 3}$.