Quiz 11
Due: 14 November 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: ____

1. Find the exact value of $\sec(\frac{3\pi}{8})$ by hand.

Solution: Since $\sec(\frac{3\pi}{8}) = 1/\cos(\frac{3\pi}{8})$, we really just need to find $\cos(\frac{3\pi}{8})$. Notice that $\frac{3\pi}{8}$ is half of $\frac{3\pi}{4}$, i.e.,

$$\frac{3\pi}{8} = \frac{\frac{3\pi}{4}}{2}.$$

So we can use the half-angle formula for cosine. Remember that

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}.$$

We choose the "+" or "−" based on whether $\cos(\frac{x}{2})$ is positive or negative, i.e., based on what quadrant the angle $\frac{x}{2}$ is in. Since $\frac{3\pi}{8}$ is in the first quadrant, we know $\cos(3\pi/8)$ is *positive*. Hence, by the formula we have

$$\cos\left(\frac{3\pi}{8}\right) = \cos\left(\frac{\frac{3\pi}{4}}{2}\right) = \sqrt{\frac{1+\cos(\frac{3\pi}{4})}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}.$$

Therefore

$$\sec\left(\frac{3\pi}{8}\right) = \frac{1}{\cos\left(\frac{3\pi}{8}\right)} = \boxed{\sqrt{\frac{2}{1-\frac{\sqrt{2}}{2}}}}.$$

2. Verify that $\dfrac{2\cos(2x)}{\sin(2x)} = \cot(x) - \tan(x)$.

Solution: We use the double angle formulas for both sine and cosine to get

$$\frac{2\cos(2x)}{\sin(2x)} = \frac{2[\cos^2(x)-\sin^2(x)]}{2\sin(x)\cos(x)} = \frac{\cos^2(x)}{\sin(x)\cos(x)} - \frac{\sin^2(x)}{\sin(x)\cos(x)} = \cot(x) - \tan(x).$$

3. Suppose that $\csc(x) = -5$ and $\pi < x < \frac{3\pi}{2}$. Compute $\sin(\frac{x}{2})$ by hand.

Solution: Since $\csc(x) = -5$ this means that $\sin(x) = -1/5$. Since x is in the third quadrant we know that $\cos(x)$ is *negative*. We find $\cos(x)$ using the Pythagorean identity:

$$\cos(x) = -\sqrt{1-\sin^2(x)} = -\sqrt{1-(-1/5)^2} = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}.$$

Lastly, we use the half-angle formula for sine. Remember that

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}.$$

We choose the "+" or "−" based on whether $\sin(\frac{x}{2})$ is positive or negative, i.e., based on what quadrant that $\frac{x}{2}$ is in. We know that $\pi < x < \frac{3\pi}{2}$. Dividing this inequality by 2 shows that $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$. Therefore $\frac{x}{2}$ is in quadrant 2 and so $\sin(x/2)$ is *positive*. Hence overall we compute

$$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-(-\frac{2\sqrt{6}}{5})}{2}} = \boxed{\sqrt{\frac{1+\frac{2\sqrt{6}}{5}}{2}}.}$$

4. Use the figure to compute each of the following:
(*You do not need to simplify your answers.*)

a.) $\tan\left(\frac{\alpha}{2}\right)$;

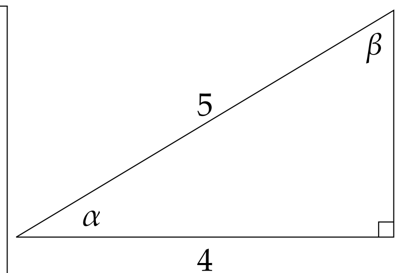
Solution: Notice $\sin(\alpha) = \frac{3}{5}$ and $\cos(\alpha) = \frac{4}{5}$. So

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin(\alpha)}{1+\cos(\alpha)} = \boxed{\frac{\frac{3}{5}}{1+\frac{4}{5}}}.$$

b.) $\cos(2\beta)$;

Solution: Notice $\cos(\beta) = \frac{4}{5}$ and $\sin(\beta) = \frac{3}{5}$. So

$$\cos(2\beta) = \cos^2(\beta) - \sin^2(\beta) = \boxed{\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2}.$$



c.) $\sin(4\alpha)$.

Solution: First observe

$$\begin{aligned}\sin(4\alpha) &= 2\sin(2\alpha)\cos(2\alpha)\\ &= 4\sin(\alpha)\cos(\alpha)[\cos^2(\alpha) - \sin^2(\alpha)].\end{aligned}$$

Hence we have that

$$\sin(4\alpha) = \boxed{4\cdot\frac{3}{5}\cdot\frac{4}{5}\left(\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2\right)}.$$