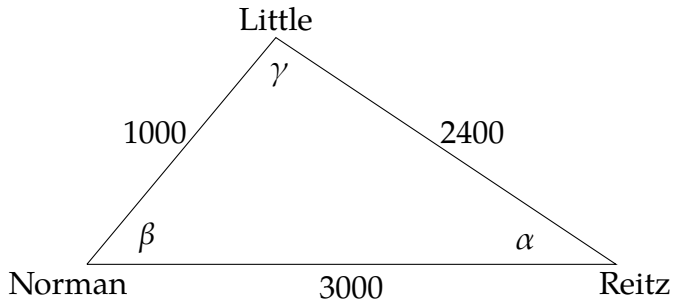
Quiz 12
Due: 21 November 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: _____

1. The Reitz Union is 2,400 feet from Little Hall, Norman Hall is 3,000 feet from the Reitz Union, and Little Hall is 1,000 feet from Norman Hall. Draw a triangle connecting these buildings and calculate the **smallest** angle in your triangle. (You may leave your answer in the form of an inverse trigonometric function.)

Solution: Using the given information we setup a triangle as below:

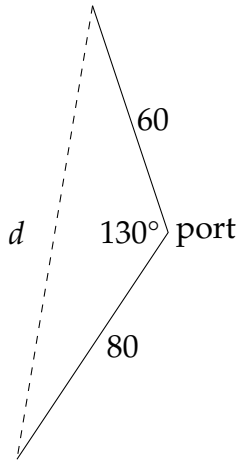


(We haven't worried about orientation or scale in our drawing.) The **smallest** angle will be the one opposite to the **shortest** side, so in this case we need to compute α. By the law of cosines,

$$\alpha = \arccos\left(\frac{3000^2 + 2400^2 - 1000^2}{2 \cdot 3000 \cdot 2400}\right) \approx 17.15^\circ.$$

2. Two boats leave the same port at the same time. One sails 30 miles per hour at a bearing of N20°W while the other sails 40 miles per hour at a bearing of S30°W. How far apart are these two boats after 2 hours?

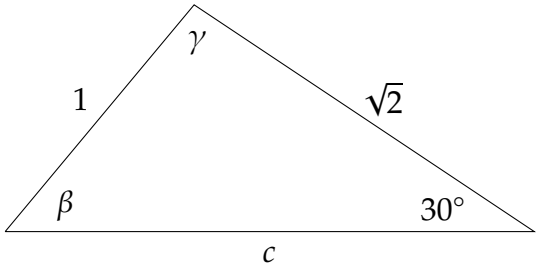
Solution: We use the given information to setup the following picture:



We wish to find the length of the dashed line d. By the law of cosines,

$$\boxed{d = \sqrt{60^2 + 80^2 - 2 \cdot 60 \cdot 80 \cdot \cos(130^\circ)} \approx 74.5 \text{ miles.}}$$

3. Determine β, γ, and c in the triangle below. Assume that β is **acute**. (Not to scale.)



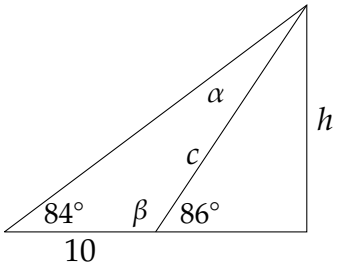
Solution: Looking at the given information in the triangle, we find it best to solve for β using the law of sines, which says that

$$\frac{\sin(\beta)}{\sqrt{2}} = \frac{\sin(30^\circ)}{1} \iff \sin(\beta) = \frac{\sqrt{2}}{2} \iff \boxed{\beta = 45^\circ.}$$

Therefore $\gamma = 180^\circ - 30^\circ - 45^\circ = \boxed{105^\circ.}$ From here we can find c in several ways. For example, by the law of sines $c = \sin(105^\circ)/\sin(30^\circ) = \boxed{2\sin(105^\circ).}$

4. You and a friend are watching a bird flying directly overhead. When the bird is to the right of you and your friend, you measure the angle of elevation to the bird to be 84° while your friend measures the angle of elevation to the bird to be 86°. If you and your friend are standing 10 feet apart, what is the altitude of the bird in feet?

Solution: We use the given information to setup the following triangle:



The goal is to find h. Notice that $\sin(86°) = h/c$, so we have $h = c \cdot \sin(86°)$. Now we need to determine c. We can use the law of sines to compute c once we know what α is. But $\beta = 180° - 86° = 94°$ which implies $\alpha = 180° - 84° - 94° = 2°$. Hence

$$\frac{c}{\sin(84°)} = \frac{10}{\sin(2°)} \iff c = \frac{10\sin(84°)}{\sin(2°)}.$$

Finally we have that the altitude of the bird is

$$h = c \cdot \sin(86°) = \boxed{\frac{10\sin(84°)\sin(86°)}{\sin(2°)}.}$$