Quiz 2
Due: 10 September 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: ______________________________

1. Demonstrate how to use the unit circle and coterminal angles to calculate

$$\sin(-840°) \cdot \tan\left(\frac{17\pi}{6}\right) + \cot\left(-\frac{3\pi}{4}\right) \cdot \cos\left(\frac{\pi}{3}\right)$$

by hand.

Solution: Notice that $-840°$ is coterminal to $-120°$, from which we can use the unit circle and the fact that $\sin(-\theta) = -\sin(\theta)$ to compute

$$\sin(-840°) = \sin(-120°) = -\sin(120°) = -\frac{\sqrt{3}}{2}.$$

Likewise, $\frac{17\pi}{6}$ is coterminal to $\frac{5\pi}{6}$ and thus

$$\tan\left(\frac{17\pi}{6}\right) = \tan\left(\frac{5\pi}{6}\right) = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}.$$

Combining these two results shows that the first term above is simply

$$\sin(-840°) \cdot \tan(17\pi/6) = \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{3}}\right) = \frac{1}{2}.$$

For the second term, we again use the unit circle and $\cot(-\theta) = -\cot(\theta)$ to find

$$\cot\left(-\frac{3\pi}{4}\right) = -\cot\left(\frac{3\pi}{4}\right) = -\left(\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right) = 1 \quad \text{and} \quad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}.$$

Hence $\cot(-3\pi/4) \cdot \cos(\pi/3) = 1 \cdot \frac{1}{2} = \frac{1}{2}$ and so overall we have

$$\sin(-840°) \cdot \tan\left(\frac{17\pi}{6}\right) + \cot\left(-\frac{3\pi}{4}\right) \cdot \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{1}{2} = \boxed{1.}$$

2. Show how to determine which is bigger: $\tan(\cos(\frac{\pi}{2}))$ or $\cos(\sin(\pi))$?

Solution: We use the unit circle to evaluate these compositions of functions as

$$\tan(\cos(\pi/2)) = \tan(0) = 0 \quad \text{and} \quad \cos(\sin(\pi)) = \cos(0) = 1.$$

Since $0 < 1$ we conclude that $\boxed{\cos(\sin(\pi))}$ is bigger than $\tan(\cos(\frac{\pi}{2}))$.

3. Suppose that $\tan(\theta) = \sqrt{3}/3$. Give at least three different possibilities for the angle θ.

$$\left(\text{Hint: recall that } \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2}.\right)$$

Solution: Following the hint, we are looking for angles θ where $\sin(\theta) = 1/2$ and $\cos(\theta) = \sqrt{3}/2$, or where $\sin(\theta) = -1/2$ and $\cos(\theta) = -\sqrt{3}/2$. There are infinitely many such angles. We can list a few of them as

$$\boxed{\ldots, \frac{-11\pi}{6}, \frac{\pi}{6}, \frac{13\pi}{6}, \ldots \quad \text{and} \quad \ldots, \frac{-5\pi}{6}, \frac{7\pi}{6}, \frac{19\pi}{6}, \ldots}$$

or write them all as $\boxed{\left\{\frac{\pi}{6} + 2\pi k : k \text{ is an integer}\right\} \cup \left\{\frac{7\pi}{6} + 2\pi k : k \text{ is an integer}\right\}}$.

(I typically abbreviate the set notation to $\{\pi/6 + 2\pi k\} \cup \{7\pi/6 + 2\pi k\}$.)

4. Suppose that $\cos(\theta) = \frac{5}{13}$ and $\sin(\theta) = -\frac{12}{13}$. Show how to compute $5 \cdot \tan(-\theta)$.

Solution: We know that $\tan(-\theta) = -\tan(\theta)$ and $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$. Therefore

$$5 \cdot \tan(-\theta) = -5 \cdot \frac{\sin(\theta)}{\cos(\theta)} = -5 \cdot \frac{\frac{-12}{13}}{\frac{5}{13}} = -5 \cdot \frac{-12}{5} = \boxed{12.}$$

(It can be helpful to think about if this sign makes sense. Here $\cos(\theta)$ is positive and $\sin(\theta)$ is negative, so we know the terminal side of θ lies in quadrant four. Hence the terminal side of $-\theta$ lies in quadrant one, so $\cos(-\theta)$ and $\sin(-\theta)$ are both positive and therefore $\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$ is positive as well.)