Quiz 2
28 May 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: _____

1. Identify the traces parallel to the yz-plane for the quadric surface $z = x^2 - y^2$.

Solution: The planes parallel to the yz-plane are of the form $x = c$ for some real number c. Hence these traces are $z = c^2 - y^2$, so things like $z = 1 - y^2$ or $z = 9 - y^2$. These are $\boxed{\text{parabolas.}}$

2. Find an equation of the plane that passes through the point $(1, 2, 3)$ and is perpendicular to the line of intersection of the planes $x + y - z + 3 = 0$ and $3x - y + 2z = 0$.

Solution: The planes have normal vectors $\langle 1, 1, -1\rangle$ and $\langle 3, -1, 2\rangle$ respectively. The line of intersection of these two planes must be orthogonal to both of these normal vectors, hence it must be parallel to their cross product, which is

$$\langle 1, 1, -1\rangle \times \langle 3, -1, 2\rangle = \langle 1, -5, -4\rangle.$$

The general equation of a plane is $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$, where $\langle a, b, c\rangle$ is normal to the plane and (x_0, y_0, z_0) is a point on the plane. Hence the desired plane can be represented by the equation

$$\boxed{(x - 1) - 5(y - 2) - 4(z - 3) = 0,}$$

or equivalently, $\boxed{x - 5y - 4z = -21.}$