Quiz 3
Due: 17 September 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: ______________________________

1. Suppose θ is an acute angle with $\cos(\theta) = \frac{1}{3}$. Use this to compute $\cot(\frac{\pi}{2} - \theta)$.

Solution: First note that, using the cofunction identity, we have

$$\cot(\pi/2 - \theta) = \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}. \qquad (\ddagger)$$

We already know $\cos(\theta)$, so we just need to determine $\sin(\theta)$ and then plug these into ($\ddagger$). To determine $\sin(\theta)$ we can use the Pythagorean identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1 \quad \Longrightarrow \quad (1/3)^2 + \sin^2(\theta) = 1 \quad \Longrightarrow \quad \sin^2(\theta) = 8/9.$$

So $\sin^2(\theta) = 8/9$ and thus $|\sin(\theta)| = \sqrt{8/9} = \frac{2\sqrt{2}}{3}$. How can we be sure whether

$$\sin(\theta) = \frac{2\sqrt{2}}{3} \quad \text{or} \quad \sin(\theta) = -\frac{2\sqrt{2}}{3} \quad ??$$

Since we're told that θ is acute, we know $\sin(\theta)$ is positive. So, overall we have

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta) = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = \frac{2\sqrt{2}}{3} \cdot \frac{3}{1} = \boxed{2\sqrt{2}.}$$

2. Suppose θ is an obtuse angle with $\cot(\theta) = -\frac{12}{5}$. Use this to compute $\sin(\theta)$.

Solution: One of the Pythagorean identities says that $\cot^2(\theta) + 1 = \csc^2(\theta)$. Hence

$$\csc^2(\theta) = (-12/5)^2 + 1 = \frac{144}{25} + 1 = \frac{169}{25}.$$

Since $\csc(\theta) = 1/\sin(\theta)$ this implies $\sin^2(\theta) = \frac{25}{169}$ and so $|\sin(\theta)| = 5/13$. Since θ is obtuse we know $\sin(\theta)$ is positive, therefore we conclude $\boxed{\sin(\theta) = 5/13.}$

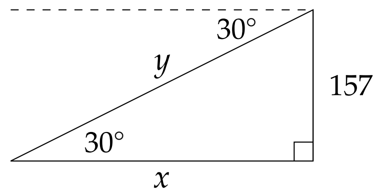
3. You want to install a zipline from the top of the Century Tower to the ground at a 30° angle of depression. The Tower is 157 feet tall.

   a.) How far away from the base of the tower will you land?

   b.) How much cable will you need?

(Give **exact** answers—no decimals.)

Solution: We model the situation with the following triangle:

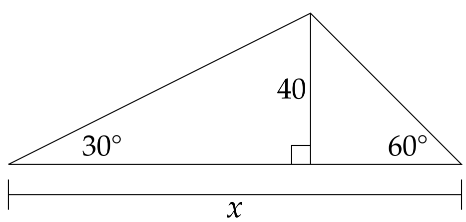


Part (a) wants us to determine x and part (b) wants us to determine y.

Notice that $\tan(30°) = 157/x$ which implies $x = 157/\tan(30°) = \boxed{157\sqrt{3} \text{ feet.}}$

Similarly $\sin(30°) = 157/y$ which implies $y = 157/\sin(30°) = \boxed{314 \text{ feet.}}$

4. Find the **exact** value of x below. (Figure not to scale.)



Solution: Let a and b be the side lengths of the left and right triangles, respectively, so that $x = a+b$. Then we have

$$\frac{40}{a} = \tan(30°) = \frac{\sin(30°)}{\cos(30°)} = \frac{1}{\sqrt{3}}$$

which shows $a = 40\sqrt{3}$. Likewise, we find that

$$\frac{40}{b} = \tan(60°) = \sqrt{3} \quad \text{and so} \quad b = \frac{40}{\sqrt{3}}.$$

Therefore $\boxed{x = 40\sqrt{3} + 40/\sqrt{3}.}$