**Quiz 4**
18 June 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: _____

1. Let $w = 2xy^2z^3$ with $x = r\cos t$, $y = t\sin r$, and $z = e^{rt}$. Find $\dfrac{\partial w}{\partial r}$ when $(r, t) = (1, 0)$.

**Solution:** Apply the chain rule and take partial derivatives to compute

$$\begin{aligned}\frac{\partial w}{\partial r} &= \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r} \\ &= (2y^2z^3)(\cos t) + (2xyz^3)(t\cos r) + (6xy^2z^2)(te^{rt}).\end{aligned}$$

At the point $(r, t) = (1, 0)$ we have that $x = 1$, $y = 0$, and $z = 1$. Observe that each term in our expression for $\frac{\partial w}{\partial r}$ above contains $y$ as a factor. Hence

$$\boxed{\frac{\partial w}{\partial r} = 0 + 0 + 0 = 0 \quad \text{when} \quad (r, t) = (1, 0).}$$

2. Find the tangent plane to the surface $\sqrt{xy + xz + yz} = 0$ at the point $(0, 2, 2)$.

**Solution:** Write $F(x, y, z) = \sqrt{xy + xz + yz}$. The equation for the tangent plane to the surface $F(x, y, z) = 0$ at the point $(0, 2, 2)$ is

$$F_x(0,2,2)(x - 0) + F_y(0,2,2)(y - 2) + F_z(0,2,2)(z - 2) = 0.$$

We compute

$$\begin{aligned}\nabla F(x, y, z) &= \big\langle F_x(x, y, z), F_y(x, y, z), F_z(x, y, z)\big\rangle \\ &= \left\langle \frac{y + z}{2\sqrt{xy + xz + yz}}, \frac{x + z}{2\sqrt{xy + xz + yz}}, \frac{x + y}{2\sqrt{xy + xz + yz}} \right\rangle.\end{aligned}$$

Hence $\nabla F(0, 2, 2) = \langle 1, \frac{1}{2}, \frac{1}{2}\rangle$ so the desired plane is given by

$$\boxed{x + \frac{y}{2} + \frac{z}{2} = 2} \quad \text{or equivalently} \quad \boxed{2x + y + z = 4.}$$

**The latter is more likely to appear as a multiple-choice answer in this class.**