Quiz 5
9 July 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: _____

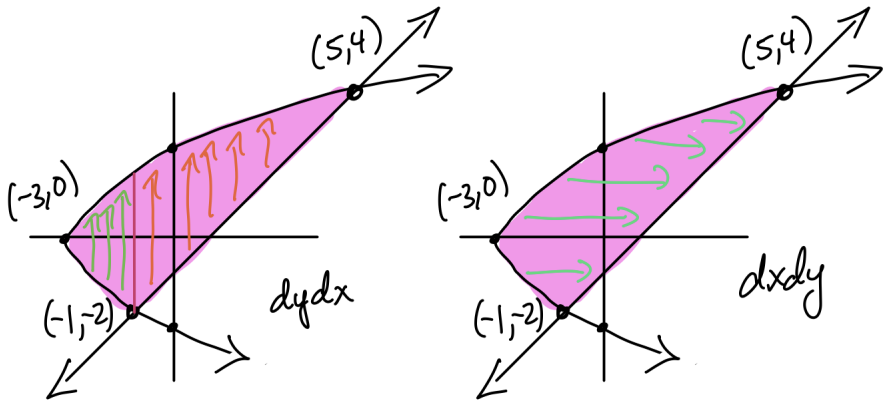
1. Let D be the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$. Sketch D in the xy-plane and then **setup** an integral to evaluate

$$\iint_D xy \, \mathrm{d}A.$$

Solution: It's crucial to sketch the region of integration for these problems. By looking at our sketch we notice that we would definitely prefer to integrate with respect to x and then y, because as x varies between -3 and 5 we have a different behavior for the y component based on whether x is less than or greater than -1. So we would need to split the integral up into two separate ones and this seems like more work. Instead, observe that whenever y varies between -2 and 4 we simply have that the x component varies between the parabola $y^2/2 - 3$ and the line $y + 1$. Hence

$$\iint_D xy \, dA = \int_{-2}^{4} \int_{\frac{1}{2}y^2-3}^{y+1} xy \, dx \, dy.$$

and you're now in a good position to evaluate the integral.



2. Find the volume of the solid S **in the first octant** that is bounded by the surface $x^2 + 2y^2 + z = 16$ and planes $x = 2$ and $y = 2$.

Solution: Since S is the solid that lies under the surface $z = 16 - x^2 - 2y^2$ and above the square $R = [0, 2] \times [0, 2]$ in the xy-plane, it follows that

$$\begin{aligned}
\text{the volume of } S &= \iint_R z \, dA \\
&= \int_0^2 \int_0^2 (16 - x^2 - 2y^2) \, dx \, dy \\
&= \int_0^2 \left[16x - \frac{x^3}{3} - 2y^2 x \right]_{x=0}^{x=2} dy \\
&= \int_0^2 \left(\frac{88}{3} - 4y^2 \right) dy \\
&= \left[\frac{88}{3} y - \frac{4}{3} y^3 \right]_{y=0}^{y=2} \\
&= \frac{176 - 32}{3} \\
&= 48.
\end{aligned}$$