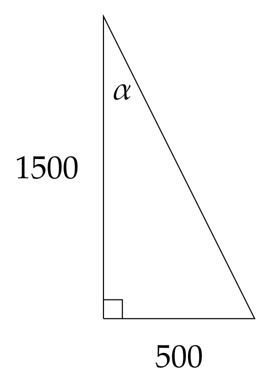
Quiz 6–7
Due: 10 October 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: ____

1. You and a friend are both at Matherly Hall leaving class. You both travel to Little Hall, but decide to take different routes. Your friend walks 1500 feet south, and then walks 500 feet east. You decide to walk in a straight line from Matherly to Little. On what bearing should you walk? Give your answer in the form S______°E.

Solution: We model this scenario with the following right triangle:



We need to find the angle α in degrees. Notice that

$$\alpha = \arctan\left(\frac{500}{1500}\right) = \arctan\left(\frac{1}{3}\right).$$

This is an exact value that we cannot compute by hand. We're not done yet though because arctan(1/3) returns a *radian* measurement of α, but we need to give our bearing in degrees. Hence we convert to degrees and our final answer is

$$\boxed{\text{S} \quad \arctan(1/3) \cdot \left(\frac{180^\circ}{\pi}\right) \quad \text{E.}}$$

2. Evaluate the following expression exactly **without a calculator**:

$$\frac{\arctan\left(\sqrt{3}\right) - \arcsin\left(\frac{\sqrt{3}}{2}\right) + \operatorname{arcsec}(-1)}{\arccos\left(\frac{1}{2}\right) - \arcsin\left(\frac{1}{2}\right)}$$

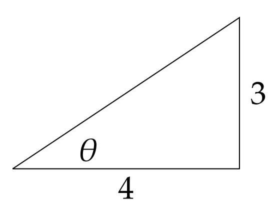
Solution: We can use the unit circle to compute each of these terms individually:

$$\begin{aligned}\frac{\arctan\left(\sqrt{3}\right) - \arcsin\left(\frac{\sqrt{3}}{2}\right) + \operatorname{arcsec}(-1)}{\arccos\left(\frac{1}{2}\right) - \arcsin\left(\frac{1}{2}\right)} &= \frac{\frac{\pi}{3} - \frac{\pi}{3} + \pi}{\frac{\pi}{3} - \frac{\pi}{6}} \\ &= \frac{\pi}{\pi/6} \\ &= \boxed{6.}\end{aligned}$$

3. Show how to compute each of the following by hand.

a.) $\cos\left(\arctan\left(\frac{3}{4}\right)\right)$;

Solution: We know that $\arctan(3/4)$ is an acute angle θ such that $\tan(\theta) = 3/4$. Hence we can draw a right triangle having θ as an angle like so:



By the Pythagorean theorem we know the hypotenuse of this triangle has length 5 and so we conclude that $\cos(\arctan(3/4)) = \cos(\theta) = \boxed{4/5.}$

b.) $\arccos(\sin(\arctan(\cos(\pi))))$;

Solution: We use the unit circle to compute this composition of functions as

$$\begin{aligned}\arccos(\sin(\arctan(\cos(\pi)))) &= \arccos(\sin(\arctan(-1))) \\ &= \arccos(\sin(-\pi/4)) \\ &= \arccos(-\sqrt{2}/2) \\ &= \boxed{3\pi/4.}\end{aligned}$$

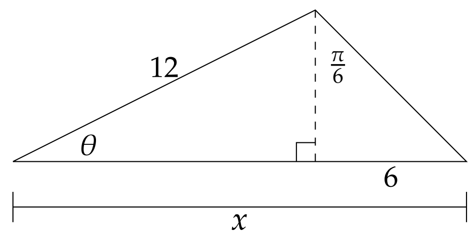
c.) $\arcsin(\sin(\frac{3\pi}{4}))$.

Solution: Again we use the unit circle to compute this composition as

$$\arcsin(\sin(3\pi/4)) = \arcsin(\sqrt{2}/2) = \boxed{\pi/4.}$$

(This is an example of an angle for which $\arcsin(\sin(\theta)) \neq \theta$.)

4. Find the exact values of θ and x in the figure below: (Figure **not** to scale.)



Solution: We first find the length h of the dotted line using the right triangle on the right side. Since $\tan(\pi/6) = 6/h$ we get that $h = 6\sqrt{3}$. Now looking at the right triangle on the left, we have the length of the opposide side to θ and the hypotenuse. Hence $\sin(\theta) = 6\sqrt{3}/12 = \sqrt{3}/2$ which implies

$$\theta = \arcsin(\sqrt{3}/2) = \boxed{\pi/3.}$$

We can use this information to find the length ℓ of the side adjacent to θ. Since $\cos(\theta) = \ell/12$ we have $\ell = 12\cos(\pi/3) = 12/2 = 6$. Therefore $x = \ell + 6 = 6 + 6 = \boxed{12.}$