**Quiz 6**

16 July 2024

> Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: ______________________________

1. Rewrite

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\, dz\, dy\, dx$$

as an equivalent iterated integral in the order $dx\, dy\, dz$.

**Solution:** Inspecting the bounds above we find the solid over which we're integrating is the portion of the first octant bounded by $z = 1 - y$ and $x = y^2$. Since we wish to rewrite this in the order $dx\, dy\, dz$, we need to consider the triangular region $D$ obtained by projecting this solid onto the $yz$-plane and represent it in the form $\{(y,z) : a \le z \le b \text{ and } h_1(z) \le y \le h_2(z)\}$. Indeed, for $z$ between 0 and 1 we want the $y$ values between 0 and $1 - z$. Hence

$$D = \{(y,z) : 0 \le z \le 1 \text{ and } 0 \le y \le 1 - z\}.$$

Now for a point $(y,z)$ in $D$, the solid contains each $x$ between 0 and $y^2$. Thus the given integral above can be rewritten as

$$\iint_D \left( \int_0^{y^2} f(x,y,z)\, dx \right) dA = \boxed{\int_0^1 \int_0^{1-z} \int_0^{y^2} f(x,y,z)\, dx\, dy\, dz.}$$

(Problem 2 on next page.)

2. Describe the solid whose volume is given by

$$\int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{16-x^2-y^2} dz\ dy\ dx$$

and calculate the volume using cylindrical coordinates.

**Solution:** Inspect the bounds of the above integral to determine that the solid of integration $E$ is the portion of a sphere with radius four centered at the origin that lies in the first quadrant.

This solid can be described in terms of cylindrical coordinates as

$$\{(r, \theta, z) : 0 \leq \theta \leq \pi/2 \text{ and } 0 \leq r \leq 4 \text{ and } 0 \leq z \leq 16 - r^2\}.$$

Hence the given integral becomes

$$\begin{aligned}\iiint_E dV &= \int_0^{\frac{\pi}{2}} \int_0^4 \int_0^{16-r^2} r\ dz\ dr\ d\theta \\ &= \int_0^{\frac{\pi}{2}} \int_0^4 16r - r^3\ dr\ d\theta \\ &= \frac{\pi}{2}\left[8r^2 - \frac{r^4}{4}\right]_{r=0}^{r=4} \\ &= \boxed{32\pi.}\end{aligned}$$