Quiz 7
30 July 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: _____

1. Calculate the work done by the force field $\vec{F}(x,y) = \langle 3y - e^{\sin x}\ ,\ 7x + \cos(y^4) + 1\rangle$ on a particle that traverses the cirle $x^2 + y^2 = 9$ exactly once counter-clockwise.

Solution: Recall that

$$\text{work} = \int_C \vec{F}d\vec{r},$$

where C is the path of the particle. Notice that C is simple, closed, and is the boundary of the disk D given by $x^2 + y^2 \leq 9$ with positive orientation. Hence by Green's theorem we have that

$$\begin{aligned}\oint_C \vec{F}d\vec{r} &= \oint_C (3y - e^{\sin x})dx + (7x + \cos(y^4) + 1)dy \\ &= \iint_D (7-3)dA \\ &= \int_0^{2\pi}\int_0^3 4r\ dr\ d\theta \\ &= \boxed{36\pi.}\end{aligned}$$

2. Find the area of the region enclosed by the curve $\vec{r}(t) = \langle \sin t \cdot \cos t\,, \sin t\rangle$ for $0 \leq t \leq \pi$.

Solution: By Green's theorem we compute

$$\text{area of } D = \iint_D dA = \oint_{\partial D} x\ dy = \int_0^\pi \sin(t)\cos^2(t)\ dt = \boxed{\frac{2}{3}.}$$