**Quiz 8**

Due: 22 October 2024

> Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: ____________________________________________

1. Verify each of the following identities:

a.) $\cos(\theta) + \tan(\theta) \cdot \sin(\theta) = \sec(\theta)$;

> **Solution:** Applying definitions and the Pythagorean identity, we find
>
> $$\begin{aligned} \cos(\theta) + \tan(\theta) \cdot \sin(\theta) &= \cos(\theta) + \frac{\sin(\theta)}{\cos(\theta)} \cdot \sin(\theta) \\ &= \frac{\cos^2(\theta) + \sin^2(\theta)}{\cos(\theta)} \\ &= \frac{1}{\cos(\theta)} \\ &= \sec(\theta) \end{aligned}$$
>
> as desired.

b.) $\dfrac{1 + \sin^2(x)}{\cos^2(x)} = 1 + 2\tan^2(x)$;

> **Solution:** We split the fraction and use definitions and one of the Pythagorean identities to see that
>
> $$\begin{aligned} \frac{1 + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} + \frac{\sin^2(x)}{\cos^2(x)} = \sec^2(x) + \tan^2(x) &= [1 + \tan^2(x)] + \tan^2(x) \\ &= 1 + 2\tan^2(x) \end{aligned}$$
>
> as desired.

c.) $\dfrac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos(\theta) - \sin(\theta)$.

> **Solution:** Recall the difference of squares factors as $a^2 - b^2 = (a - b)(a + b)$. Apply this to $a = \sin(-\theta)$ and $b = \cos(-\theta)$ to get
>
> $$\begin{aligned} \frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} &= \frac{[\sin(-\theta) - \cos(-\theta)][\sin(-\theta) + \cos(-\theta)]}{\sin(-\theta) - \cos(-\theta)} \\ &= \sin(-\theta) + \cos(-\theta) \\ &= \cos(\theta) - \sin(\theta) \end{aligned}$$
>
> as desired. The last equality follows as cosine is even and sine is odd.

2. Verify that
$$\csc(\theta) \cdot \cos(\theta) \cdot \tan(\theta) = (1 - \cos^2(\theta))(1 + \cot^2(\theta)).$$
(*Hint*: Show that both sides simplify nicely to the same number.)

> **Solution:** By definition, the left side is equivalent to
>
> $$\csc(\theta) \cdot \cos(\theta) \cdot \tan(\theta) = \frac{1}{\sin(\theta)} \cdot \cos(\theta) \cdot \frac{\sin(\theta)}{\cos(\theta)} = 1.$$
>
> Likewise, by the Pythagorean identities the right side is equivalent to
>
> $$[1 - \cos^2(\theta)][1 + \cot^2(\theta)] = \sin^2(\theta) \cdot \frac{1}{\sin^2(\theta)} = 1.$$
>
> Hence the identity in question holds.

3. Verify that
$$\frac{\sec(x) + \tan(x)}{\cot(x) + \cos(x)} = \sec(x) \cdot \tan(x).$$

> **Solution:** We'll turn everything on the left side into sine and cosine to get
>
> $$\begin{aligned} \frac{\sec(x) + \tan(x)}{\cot(x) + \cos(x)} = \frac{\frac{1}{\cos(x)} + \frac{\sin(x)}{\cos(x)}}{\frac{\cos(x)}{\sin(x)} + \cos(x)} = \frac{\frac{1+\sin(x)}{\cos(x)}}{\frac{\cos(x)[1+\sin(x)]}{\sin(x)}} = \frac{\sin(x)}{\cos^2(x)} &= \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)} \\ &= \sec(x) \cdot \tan(x) \end{aligned}$$
>
> as desired.