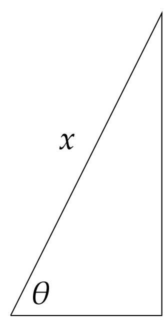
Quiz 9
Due: 5 November 2024

Answer the questions in the spaces provided. **Show all of your work and circle the answer you would like to have graded for each question.**

Name: ____

1. According to OSHA safety regulations, the proper angle for setting up a ladder is to place it 1 foot away from the wall for every 4 feet of ladder length. Find the recommended working angle for a ladder of length x. Click here for more details.
(This aims to correct an inaccurate claim that I made in class.)

Solution: Since the ladder length is four times the distance from the wall, we can setup a triangle like the one below:



Hence $\boxed{\theta = \arccos\left(\frac{\frac{1}{4}x}{x}\right) = \arccos(1/4) \approx 75.52^\circ.}$

2. Find **all** exact solutions to each of the following equations:

a.) $\sec^2 x - 4 = 0$;

Solution: Observe that

$$\sec^2(x) - 4 = 0 \iff \sec^2(x) = 4 \iff \cos^2(x) = 1/4 \iff \cos(x) = \pm 1/2.$$

Hence the solutions to the given equation are the same as the solutions to $\cos(x) = \pm 1/2$. We can use the unit circle to find that these solutions are

$$\boxed{\left\{\frac{\pi}{3} + 2\pi n\right\} \cup \left\{\frac{2\pi}{3} + 2\pi n\right\} \cup \left\{\frac{4\pi}{3} + 2\pi n\right\} \cup \left\{\frac{5\pi}{3} + 2\pi n\right\}.}$$

b.) $2\sin(\pi\theta) + 1 = 0$.

Solution: Observe that

$$2\sin(\pi\theta) + 1 = 0 \iff \sin(\pi\theta) = -\frac{1}{2},$$

so we know from the unit circle that

$$\pi\theta = \frac{7\pi}{6} + 2\pi n \quad \text{or} \quad \pi\theta = \frac{11\pi}{6} + 2\pi n.$$

Solving for θ, we determine the solutions are

$$\boxed{\left\{\frac{7}{6} + 2n\right\} \cup \left\{\frac{11}{6} + 2n\right\}.}$$

3. Find **all** exact solutions to each of the following equations:

a.) $\cos^3(x) = \cos(x)$;

Solution: We can move everything to one side and factor to get

$$\cos(x) - \cos^3(x) = 0 \iff \cos(x)[1 - \cos^2(x)] = 0 \iff \cos(x)\sin^2(x) = 0.$$

Hence the solutions to the given equation are the same as the ones for $\cos(x) = 0$ or $\sin^2(x) = 0$. For the former, the unit circle tells us the solutions are $x = \frac{\pi}{2} + \pi n$. For the latter, $\sin^2(x) = 0$ is equivalent to $\sin(x) = 0$, whose solutions are $x = \pi n$. So the overall set of solutions is

$$\boxed{\{\pi/2 + \pi n\} \cup \{\pi n\}.}$$

b.) $\tan(x) = 3\sin(x)$;

Solution: Similar to the last problem, we move everything to one side and factor to get

$$\tan(x) - 3\sin(x) = 0 \iff \sin(x)[\sec(x) - 3] = 0.$$

Hence the solutions to the given equation are those for $\sin(x) = 0$ or $\sec(x) = 3$. We computed the former in the previous problem. For the latter, this is equivalent to $\cos(x) = 1/3$. The solutions to this equation are

$$x = \arccos(1/3) + 2\pi n \quad \text{or} \quad x = -\arccos(1/3) + 2\pi n.$$

Hence our final answer is

$$\boxed{\{\pi n\} \cup \{\arccos(1/3) + 2\pi n\} \cup \{-\arccos(1/3) + 2\pi n\}.}$$

c.) $2\cos(3\theta) = \sqrt{2}$;

Solution: Observe that

$$2\cos(3\theta) = \sqrt{2} \iff \cos(3\theta) = \sqrt{2}/2.$$

This happens when 3θ is coterminal to either $\pi/4$ or $7\pi/4$. Hence our final answer is

$$\boxed{\left\{\frac{\pi}{12} + \frac{2\pi}{3}n\right\} \cup \left\{\frac{7\pi}{12} + \frac{2\pi}{3}n\right\}.}$$

4. Find the exact solution(s) between 0 and 2π satisfying $\cos^2(x) = 2\sin(x) + 2$.

Solution: Use the Pythagorean identity to replace $\cos^2(x)$ by $1 - \sin^2(x)$ so that we get the quadratic equation

$$1 - \sin^2(x) = 2\sin(x) + 2.$$

Move everything to one side and factor to get

$$\sin^2(x) + 2\sin(x) + 1 = 0 \iff (\sin(x) + 1)^2 = 0.$$

Hence the solutions to the original equation are the same as the solutions to $\sin(x) + 1 = 0$. This is equivalent to $\sin(x) = -1$. We use the unit circle to find the solutions

$$\left\{\frac{3\pi}{2} + 2\pi n\right\}.$$

The only solution between 0 and 2π is $\boxed{3\pi/2.}$