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Xronos 05 #13

We want the plane P which passes through the intersection of the planes

$P_1 : x - z + 2 = 0$ and perpendicular
$P_2 : y + 2z - 2 = 0$, to $P_3 : x+y+4z=3$.

Any such plane passing through the intersection of P_1 and P_2 is of the form

$$x - z + 2 + k(y + 2z - 2) = 0 \text{ for some } k.$$

Equivalently, $x + ky + (2k-1)z + 2(1-k) = 0$.
This has normal vector $\langle 1, k, 2k-1 \rangle$.
Since we want to be perpendicular to P_3, we need $\langle 1, k, 2k-1 \rangle \cdot \langle 1, 1, 4 \rangle = 0$.
So then

$$0 = 1 + k + 4(2k-1) = 9k - 3$$

and therefore $k = 1/3$. So the equation for P is

$$x + \tfrac{1}{3}y + (2\cdot\tfrac{1}{3} - 1)z + 2(1 - \tfrac{1}{3}) = 0,$$

which is the same as $\boxed{3x + y - z + 4 = 0}$.