Announcements: · Exam 3 is graded, you'll get them back @ the end of class, along w/ quiz 6 if you didn't get it last week. VERIFY GRADES

· Final Exam is Dec 9 @ 5:30 PM, all multiple choice!

· Questions can be answered during office hrs Thurs 3-5

Today's Quiz: Potential Functions & Vector Line Integrals

L27 - Vector Fields - funky graphs of vectors

Conservative vector fields: fields generated by taking the gradient of a scalar valued function

The scalar valued function is called a potential function.

Practice in 2D:

X27 #7:

"derivative"

Find the potential function f such that $F = \nabla f$.

$$\rightarrow F(x,y) = 9ye^{9x}\,i + (e^{9x} + 4y^3)\,j$$

$\int F = \int \cancel{\nabla} f$

$\int F = f$

We want to get a function f, so that when we take the derivative of f, we get back F.

So we integrate F w.r.t. one variable, say x:

$$f(x,y,z) = \int 9ye^{9x}\,dx = 9y\int e^{9x}\,dx = \cancel{9}y\left(\frac{1}{\cancel{9}}e^{9x}\right) = ye^{9x} + g(y) + c$$

$\nabla f = \langle f_x, f_y \rangle$

Now we compare f_y to the y component of F:

$$f_y = \cancel{e^{9x}} + g'(y) = \cancel{e^{9x}} + 4y^3$$

$$\int g'(y) = \int 4y^3$$

$$g(y) = y^4 + c$$

$$f(x,y) = ye^{9x} + y^4 + c$$

$$F(x,y) = 9ye^{9x}\,i + (e^{9x} + 4y^3)\,j$$

$$\int 9ye^{9x}\,dx = ye^{9x} + c \qquad \int (e^{9x} + 4y^3)\,dy = ye^{9x} + y^4 + c$$

$$f(x,y,z) = ye^{9x} + y^4 + c$$

Practice in 3D:

X 27 #12

$f(x,y,z) = 2z + h(x) + g(y)$

Find a function f such that $\vec{F} = \nabla f$. ***

$F(x,y,z) = (xy\cos(xy) + \sin(xy))i + x^2\cos(xy)j + 2k$

First integrate one component: [y is easiest here]

$f(x,y,z) = \int x^2\cos(xy)\,dy = x^2\int \cos(xy)\,dy = x^2 \cdot \frac{1}{x}\sin(xy)$

$f(x,y,z) = x\sin(xy) + g(x) + h(z)$

Compare f_x to x component:

$f_x = \cancel{\sin(xy)} + xy\cancel{\cos(xy)} + g'(x) + 0 = xy\cancel{\cos(xy)} + \cancel{\sin(xy)}$

$\int g'(x) = \int 0$

$g(x) = c$

Now compare f_z to z component:

$f_z = \int h'(z) = \int 2$

$h(z) = 2z + c$

$$\boxed{f(x,y,z) = x\sin(xy) + 2z + c}$$:)

L28 - Line Integrals!

Scalar Line Integrals: $\displaystyle\int_a^b \underline{f(x(t), y(t))}\, |r'(t)|\, dt$

scalar function in integrand

where $r(t)$ is the parameterized curve

$a \le t \le b$

Vector Line Integrals: $\displaystyle\int_a^b \underline{\vec{F}(r(t))} \cdot r'(t)\, dt$

vector $\qquad \vec{F} = \langle x, y, z \rangle$

Practice - X28 #2

original: Let C be the part of the circle $x^2 + y^2 = 4$ in the 1st quadrant oriented counterclockwise. Evaluate $\displaystyle\int_C xy^2\, ds$.

$\vec{F}$

vector

$\langle x, y^2 \rangle$

$|r'(t)|$

scalar line integral

$x = 2\cos(t)$

$y = 2\sin(t)$

$r(t) = \langle 2\cos(t), 2\sin(t) \rangle$

$r'(t) = \langle -2\sin(t), 2\cos(t) \rangle$

$|r'(t)| = \sqrt{4\sin^2(t) + 4\cos^2(t)}$

$= \sqrt{4(\sin^2(t) + \cos^2(t))} = \sqrt{4}$

$= 2$

$$\int 2\cos(t)\,(2\sin(t))^2\, |r'(t)|$$

$$\int_0^{\pi/2} 2\cos(t)\, 4\sin^2(t)\, 2\, dt$$

$$= \int_0^{\pi/2} 16\cos(t)\sin^2(t)\, dt$$

$u = \sin(t)$

$du = \cos(t)$

$=$

Now as vector line integral: ***

Let C be the part of the circle $x^2+y^2=4$ in the $\boxed{1^{st} \text{ quadrant}}$ oriented counterclockwise. Evaluate $\int_C \vec{F}\cdot d\underline{r}$

where $\underline{\vec{F}} = \langle y, -x\rangle$

$x = 2\cos(t)$

$y = 2\sin(t)$

clockwise

$-2\sin(t)$

$\vec{F} = \langle 2\sin(t), -2\cos(t)\rangle$

$$\int_0^{\pi/2} \langle 2\sin(t), -2\cos(t)\rangle \cdot \langle -2\sin(t), 2\cos(t)\rangle dt$$

$dr = r'(t) = \langle -2\sin(t), 2\cos(t)\rangle$

$$= \int_0^{\pi/2} -4\sin^2(t) - 4\cos^2 t \, dt$$

$$= \int_0^{\pi/2} -4\left(\sin^2(t)+\cos^2(t)\right)dt$$