Announcements: Exam 3 (L19-26) next Wednesday @ 8:30PM in FLINT 0050. We can probably do a zoom review for this one ⌣

Office Hours: Halloween 3-5! There will be candy and I will be dressed up.

L22 Practice: ★★★ (From L22 page 5)

Let R be the region bounded by the lines:
$x+y=1$, $x+y=4$, $x-2y=0$, and $x-2y=-4$.

Using a transformation, evaluate the integral:

$$\iint_R 3(x+y)\,dA.$$

Set $u=x+y$, then $1 \le u \le 4$

set $v=x-2y$, then $-4 \le v \le 0$.

Find Jacobian: $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}^{-1} = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}^{-1} = (-2-1)^{-1} = -\frac{1}{3}.$

Take abs value $\Rightarrow$ $|J| = \frac{1}{3}$

Rewrite integral: $\int_{-4}^{0}\int_{1}^{4} 3u\left(\frac{1}{3}\right) du\,dv$

$$= \int_{-4}^{0} \frac{u^2}{2}\Big|_1^4 \, dv = \int_{-4}^{0} \frac{16}{2} - \frac{1}{2} \, dv = \frac{15}{2} v \Big|_{-4}^{0}$$

$$= 30$$

L23 Practice ★★★ X23 #5

Use a double integral to find the area of the region inside the circle ① $(x-5)^2 + y^2 = 25$ and outside the circle ② $x^2 + y^2 = 25$.

① Convert to polar: ①: $x^2 - 10x + 25 + y^2 = 25$

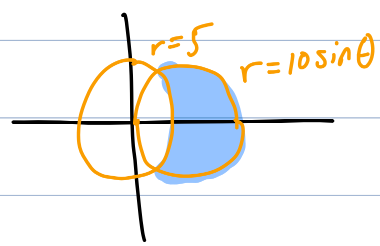
$\Rightarrow x^2 + y^2 = 10x$

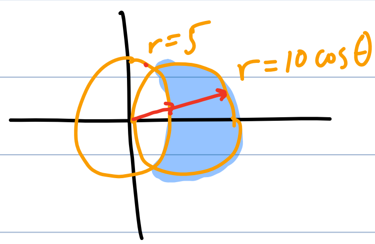
$\Rightarrow r^2 = 10r\cos\theta$

$\Rightarrow r = 10\cos\theta$

② $r^2 = 25 \Rightarrow r = 5$

② Sketch region!!





③ Consider bounds for r & θ:

$5 \le r \le 10\cos\theta$

Now find POIs for θ:

$$5 = 10\cos\theta$$

$$\Rightarrow \theta = -\pi/3, \pi/3$$

$$\int_{-\pi/3}^{\pi/3}\int_{5}^{10\cos\theta} r\,dr\,d\theta$$

$$\frac{r^2}{2}\Big|_{5}^{10\cos\theta}$$

$$50\cos^2\theta - \frac{25}{2}$$

$$= 25(1+\cos 2\theta) - \frac{25}{2}$$

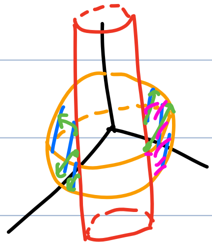
$$\Rightarrow 25\theta + \frac{25}{2}\sin 2\theta - \frac{25}{2}\theta \Big|_{-\pi/3}^{\pi/3}$$

$$\frac{25\pi}{3} + \frac{25}{2}\frac{\sqrt{3}}{2} - \frac{25}{2}\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{25\pi}{3} + \frac{25\sqrt{3}}{2}$$

X23 #8 Use polar coordinates to find the volume of the given solid. Inside the sphere $x^2+y^2+z^2=64$ and outside the cylinder $x^2+y^2=25.$

sketch:



$\Rightarrow$ polar: $r^2+z^2=64$

$\Rightarrow z=\sqrt{64-r^2}$

will only get positive

$$2\int_0^{2\pi}\int_5^8 \sqrt{64-r^2}\; r\,dr\,d\theta$$

u-sub $u=64-r^2$

$du=-2r\,dr \quad \Rightarrow -\frac{1}{2}\int_0^{2\pi}\int \sqrt{u}\,du\,d\theta$

$-\frac{du}{2}=r\,dr$

$= \frac{2}{3}u^{3/2}$

$= \frac{2}{3}(64-r^2)^3\Big|_5^8$

$$-\frac{1}{3}\int_0^{2\pi} -(39)^{3/2}\,d\theta$$

$$2\left(\frac{2\pi}{3}39\sqrt{39}\right) = \frac{52\pi}{3}\sqrt{39}$$

10. Exam 3 2018

10. Rewrite $\int_0^{16}\int_{\sqrt{x}}^{4}\int_0^{4-y} dz\,dy\,dx$ as an equivalent iterated integral in the order $dy\,dx\,dz$.

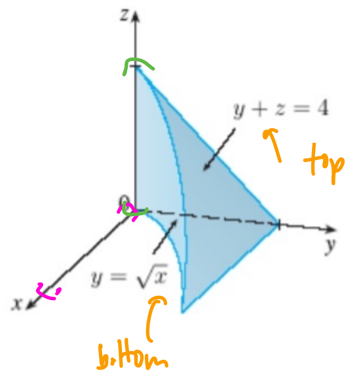
a. $\int_0^{16}\int_0^{(4-z)^2}\int_{\sqrt{x}}^{4-z} dy\,dx\,dz$

b. $\int_0^{4}\int_0^{(4-z)^2}\int_{\sqrt{x}}^{4-z} dy\,dx\,dz$

c. $\int_0^{16}\int_0^{4-\sqrt{z}}\int_0^{4-z} dy\,dx\,dz$

d. $\int_0^{4}\int_0^{4-\sqrt{z}}\int_0^{4-z} dy\,dx\,dz$

e. $\int_0^{4}\int_0^{4-z}\int_0^{\sqrt{x}} dy\,dx\,dz$



$$\int_0^{(4-z)^2}\int_{\sqrt{x}}^{4-z} dy\,dx\,dz$$

The first variable we write between surfaces (two variables)
The second variable we write btwn curves (one variable)
The third between constants (no variables)

x varies from 0 to $x = y^2$
however, we already restricted y.
so $x = (4-z)^2$

Now z goes from 0 to $4-y$, y at the least is $\sqrt{x}$, which at the least is 0, so z is max 4.

11. Which of the following represents the area of the region that lies inside $r = \sqrt{3}\sin\theta$ and outside $r = \cos\theta$?

a. $\displaystyle\int_{\pi/6}^{\pi}\int_{\cos\theta}^{\sqrt{3}\sin\theta} r\,dr\,d\theta$

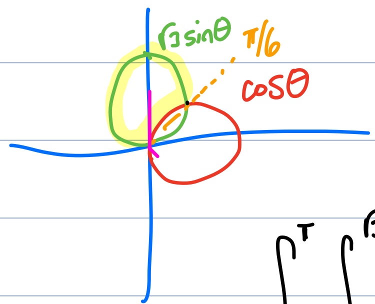
b. $\displaystyle\int_{\pi/3}^{\pi}\int_{\cos\theta}^{\sqrt{3}\sin\theta} r\,dr\,d\theta$

c. $\displaystyle\int_{\pi/3}^{\pi}\int_{0}^{\sqrt{3}\sin\theta} r\,dr\,d\theta + \int_{\pi/3}^{\pi/2}\int_{0}^{\cos\theta} r\,dr\,d\theta$

d. $\displaystyle\int_{\pi/3}^{\pi}\int_{0}^{\sqrt{3}\sin\theta} r\,dr\,d\theta - \int_{\pi/3}^{\pi/2}\int_{0}^{\cos\theta} r\,dr\,d\theta$

e. $\displaystyle\int_{\pi/6}^{\pi}\int_{0}^{\sqrt{3}\sin\theta} r\,dr\,d\theta - \int_{\pi/6}^{\pi/2}\int_{0}^{\cos\theta} r\,dr\,d\theta$

sketch:



$$\int_{\pi/6}^{\pi}\int_{0}^{\sqrt{3}\sin\theta} r\,dr\,d\theta - \int_{\pi/6}^{\pi/2}\int_{0}^{\cos\theta} r\,dr\,d\theta$$

$\cos\theta = \sqrt{3}\sin$

$\frac{1}{\sqrt{3}} = \tan\theta$

$\frac{\sqrt{3}}{3} = \tan\theta$

$\theta = \pi/6$