Announcements: When you receive your exam, make sure you ask any questions regarding your grade by next Tues, Oct 29.

Formulas to know:

Cylindrical to rectangular:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

Rectangular to cylindrical:

$$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x} \qquad z = z$$

Spherical to rectangular: ***

$$x = \rho\sin\phi\cos\theta \qquad y = \rho\sin\phi\sin\theta \qquad z = \rho\cos\phi$$

Rectangular to spherical:

$$\rho = \sqrt{x^2 + y^2 + z^2} \qquad \tan\theta = \frac{y}{x} \qquad \cos\phi = \frac{z}{\rho}$$

Quick Coordinate Practice: ***

X21 #7 Change the points from spherical coordinates (ρ, θ, φ) to rectangular.

a) $(9, \frac{\pi}{6}, \overset{\varphi}{\frac{\pi}{3}})$

$x = \rho\sin\varphi\cos\theta \qquad y = \rho\sin\varphi\sin\theta \qquad z = \rho\cos\varphi$

$x = 9\sin(\pi/3)\cos(\pi/6)$ $\qquad$ $y = 9\sin(\pi/3)\sin(\pi/6)$ $\qquad$ $z = 9\cos(\pi/3)$

$x = 9(\sqrt{3}/2)(\sqrt{3}/2)$ $\qquad$ $y = 9(\sqrt{3}/2)(1/2)$ $\qquad$ $z = 9(1/2)$

$x = 27/4$ $\qquad$ $y = 9\sqrt{3}/4$ $\qquad$ $z = 9/2$

b) $(3, \pi/2, 3\pi/4)$

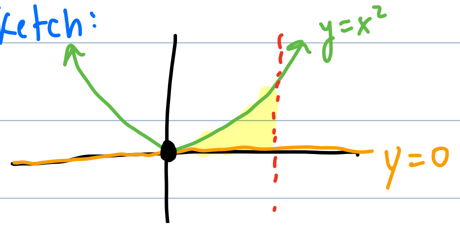
Double Integrals over General Regions:

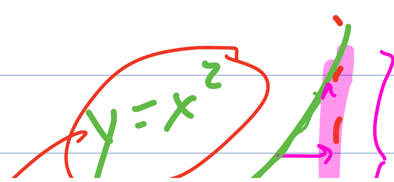
x20 #3: Evaluate the double integral **

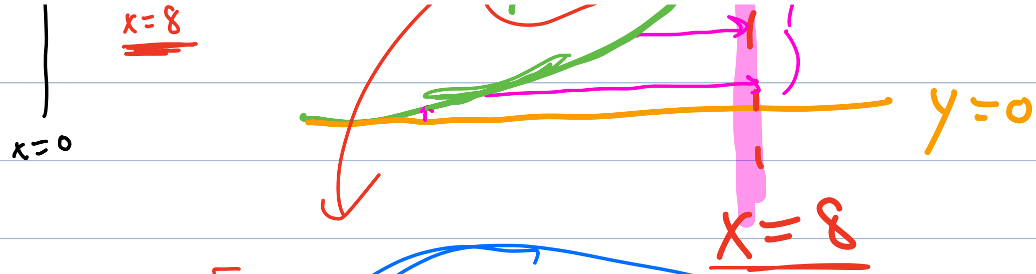
$$\iint_D \underline{7x\cos(y)}\,dA$$

where D is bounded by $y=0$, $y=x^2$, and $x=8$.

sketch:







Restrict y $\qquad \sqrt{y} = x$

$0 \le y \le x^2$

$0 \le x \le 8$

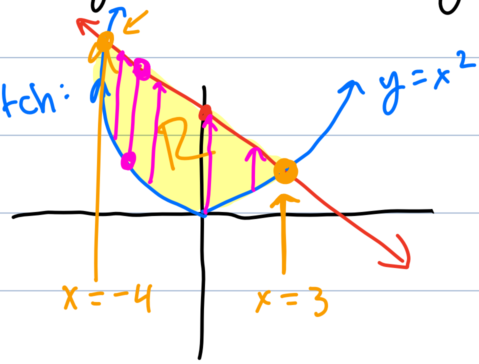
$$\int_0^8 \int_0^{x^2} 7\cos(y)\, dy\, dx$$

Restrict x

$\sqrt{y} \le x \le 8$

Let R be the region bounded by $y = x^2$ *** and $y = -x + 12$. Integrate $f(x,y) = 2x - y$ over R.

sketch:



$x^2 \le y \le -x + 12$

$-4 \le x \le 3$

Find the points of intersection:

$x^2 = -x + 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4$ and 3

$$\int_{-4}^{3} \int_{x^2}^{-x+12} 2x - y \, dy \, dx$$

$$\int_{-4}^{3} \left[2xy - \frac{y^2}{2} \Big|_{x^2}^{-x+12} \right] dx$$

$$\int_{-4}^{3} 2x(-x+12) - \frac{(-x+12)^2}{2} - \left(2x(x^2) - \frac{(x^2)^2}{2} \right) dx$$

$$\int_{-4}^{3} -2x^2 + 24x - \frac{(x^2 - 24x + 144)}{2} - \left(2x^3 - \frac{x^4}{2} \right) dx$$

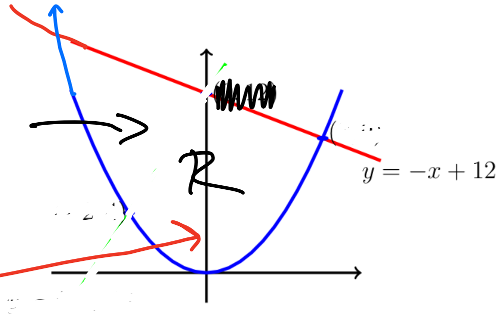
$$\boxed{\frac{-2x^3}{3} + \frac{24x^2}{2} - \frac{1}{2}\left(\frac{x^3}{3} - \frac{24x^2}{2} + 144x \right) - \left(\frac{2x^4}{4} - \frac{x^5}{2 \cdot 5} \right) \Bigg|_{-4}^{3}}$$

ex. ~~Set up a double integral that represents the area of the~~ region bounded by $y = x^2$, $y = -x + 12$, and $\cancel{y = 4x + 12}$.

$f(x,y)$

$x^3 + 2y^2$

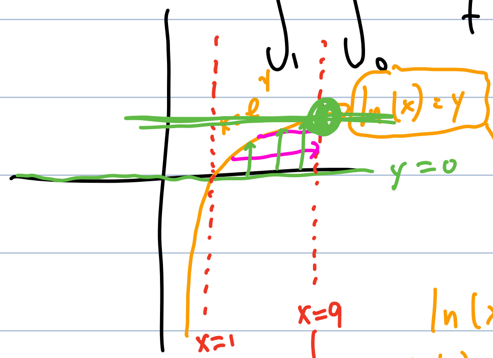
$\iint_R$



Switching Bounds:

X20 #7: Sketch the region of integration first & then change the order of integration:

$$\int_1^9 \int_0^{\ln(x)} f(x,y)\, dy\, dx$$



Originally: $0 \le y \le \ln(x)$

Restrict $1 \le x \le 9$

$e^y \le x \le 9$

$0 \le y \le \ln(9)$

$\ln(x) = y$

$e^{\ln(x)} = e^y$

$x = e^y$

$x = 9$

$e^y = 9$

$\ln(e^y) = \ln(9)$

$y = \ln(9)$

X20 #8: Sketch the region of integration first & then change the order of integration:

$$\int_{-9}^{9} \int_{0}^{\sqrt{81-x^2}} f(x,y)\, dy\, dx$$

X20 #11 Evaluate the integral by reversing the order of integration:

$$\int_0^5 \int_x^5 2e^{x/y}\, dy\, dx$$