Xronos 1 #5

Find a unit vector that is in the same direction of the vector $u - v$, if $u = \langle -3, -6, 7 \rangle$ and $v = \langle -2, 2, 3 \rangle$

step 1: Find $u - v$: $\langle -3 - (-2), -6 - 2, 7 - 3 \rangle = \langle -1, -8, 4 \rangle$

step 2: Find unit vector using $\hat{u} = \frac{u}{\|u\|}$. $\|u\| = \sqrt{(-1)^2 + (-8)^2 + 4^2} = \sqrt{81} = 9$

So the unit vector is $\frac{1}{9}\langle -1, -8, 4 \rangle$.

Q: What if we had asked for the opposite direction?

A: multiply by (-1) $\Rightarrow$ $\frac{1}{9}\langle 1, 8, -4 \rangle$.

Q: What if I wanted this vector to have a magnitude of 3?

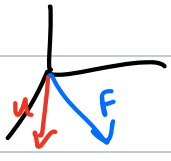
A: multiply by 3 $\Rightarrow$ $\frac{3}{9}\langle 1, 8, -4 \rangle$

X3 #7 Let $u = \langle 4, 2, -4 \rangle$. Decompose $F = \langle 6, 4, 0 \rangle$ into two vectors F_1 and F_2, where F_1 is parallel to u and F_2 is orthogonal to u.

Q: What's another way of saying F_2 is orthogonal to u?

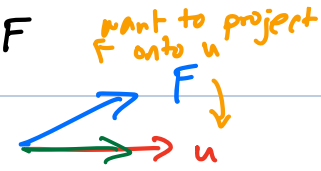
A: $F_2 \cdot u = 0$

step 0 (optional): Draw a picture



$= F_{11}$

step 1: use $\text{proj}_{\vec{u}} F = \frac{F \cdot u}{|u|^2}\vec{u}$ to find the projection of F onto u, or F's "shadow" in the direction of u



This is the vector parallel to u.

$$\Rightarrow \frac{24+8}{(\sqrt{16+4+16})^2}\langle 4,2,-4\rangle = \frac{32}{36}\langle 4,2,-4\rangle = \frac{8}{9}\langle 4,2,-4\rangle$$

step 2:

Now since $F = F_{\parallel} + F_{\perp}$ and we have $F_{\parallel}$, we just take $F - F_{\parallel} = F_{\perp}$

$$F_{\perp} = \langle 6,4,0\rangle - \frac{8}{9}\langle 4,2,-4\rangle$$

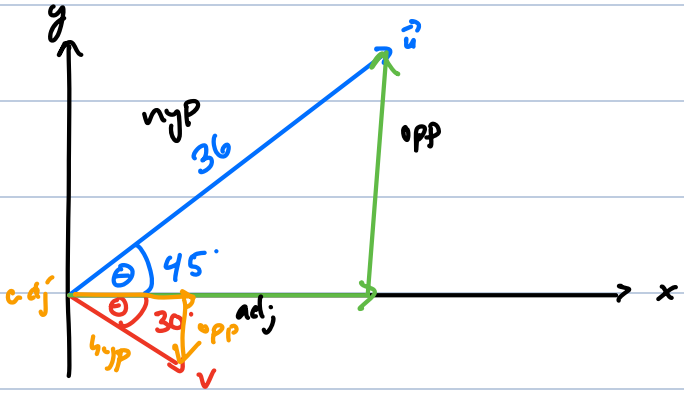
Reminders: We do not usually hone in on your algebra. Unsimplified algebraic answers are okay. Not precalc/calc ones, however.

Example: $\sqrt{52}$ is okay but not $\sin(\pi/3)$

Other FAQs:

X1 #9

SOH CAH TOA



Break into horizontal vs. vertical components

①: $u = \langle 36\cos(45^\circ), 36\sin 45\rangle$

* Xronos does not accept in

$v = \langle 20\cos(30),\ -20\sin(30)\rangle$

degree form, but radians will work: $u = \langle 36\cos(\pi/4),\ 36\sin(\pi/4)\rangle$

$v = \langle 20\cos(\pi/6),\ -20\sin(\pi/6)\rangle$

X2 #2

Which are parallel to $\langle 1, 2, -3\rangle = u$

$\langle 1, 2, 3\rangle$ ✗

$\langle -1, -2, 3\rangle$ ✓ yes $= (-1)u$

$\langle -2, -4, 6\rangle$ ✓ yes $= (-2)u$

$\langle \frac{1}{2}, 1, -\frac{3}{2}\rangle$ ✓ yes $= \frac{1}{2}(u)$

$\langle 1, 3, -3\rangle$ ✗

X3 #5 Acute angle btwn $7x - y = 2$ $6x + y = 5$

① slope-int form $y = 7x - 2$ $y = -6x + 5$

② Make vectors unit $\langle 1, 7\rangle$ $\langle 1, -6\rangle$

For every 1 unit inc. in x, y inc. 7 units

③ Use angle formula: $\cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right) = \cos^{-1}\left(\frac{1 - 42}{\sqrt{50}\sqrt{37}}\right)$

$= \cos^{-1}\left(\frac{-41}{5\sqrt{74}}\right)$

The "acuteness" here is not clear from context, but it would be on an exam/quiz.

ex: $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} = 120°$ so the

right answer here would be 180 - 120 = 60