MAC 2313 Exam 1A, Part II Free Response

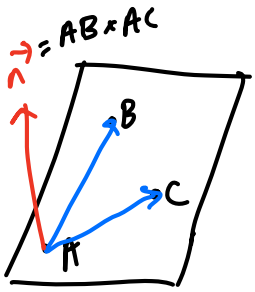
Name: _____

TA's Name: _____ Discussion Period: _____

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (10 points) Let P be the plane containing three points $A(2, 0, -1)$, $B(3, 1, 1)$ and $C(4, -1, 2)$.

(a) Find a linear equation of the plane P.



$AB = \langle 1, 1, 2 \rangle$

$AC = \langle 2, -1, 3 \rangle$

$$\begin{vmatrix} i & j & k & i & j \\ 1 & 1 & 2 & 1 & 1 \\ 2 & -1 & 3 & 2 & -1 \end{vmatrix}$$

$3i + 4j - k - 3j + 2i - 2k = \langle 5, 1, -3 \rangle$ $\vec{n}$

Now take any point, say $A = (2, 0, -1)$

$0 = \langle 5, 1, -3 \rangle \cdot \langle x-2, y-0, z-(-1) \rangle$

$0 = 5x - 10 + y - 3z - 3 \Rightarrow \underline{5x + y - 3z = 13}$

(b) Find the area of the triangle ABC.

$$= \frac{|AB \times AC|}{2} = \frac{1\sqrt{25+1+9}|}{2} = \frac{\sqrt{35}}{2}$$

2. (10 points) Let C be a smooth curve parametrized by $\vec{r}(t) = \langle \cos(t^2), \sin(t^2), t^2 \rangle$ for $t > 0$, find

(a) a unit tangent vector of the curve C

$\hat{T} = \dfrac{r'(t)}{|r'(t)|}$ $\quad r'(t) = \langle -2t\sin(t^2), 2t\cos(t^2), 2t \rangle$

$|r'(t)| = \sqrt{4t^2\sin^2(t^2) + 4t^2\cos^2(t^2) + 4t^2}$

$= \sqrt{4t^2(s^2+c^2) + 4t^2} = \sqrt{8t^2} = 2\sqrt{2}t$

$\hat{T} = \dfrac{1}{2\sqrt{2}t}\langle -2t\sin(t^2), 2t\cos(t^2), 2t \rangle = \dfrac{1}{\sqrt{2}}\langle -\sin(t^2), \cos(t^2), 1 \rangle$

$\hat{T}(t) =$ _____

(b) curvature of the curve C

$\kappa = \dfrac{|T'|}{|v|}$ $\quad T' = \dfrac{1}{\sqrt{2}}\langle -2t\cos(t^2), -2t\sin(t^2), 0 \rangle$

$|T'| = \dfrac{1}{\sqrt{2}}\sqrt{4t^2\cos^2(t^2) + 4t^2\sin(t^2)} = \dfrac{1}{\sqrt{2}}\sqrt{4t^2} = \dfrac{1}{\sqrt{2}}2t$

$\Rightarrow \dfrac{\frac{1}{\sqrt{2}}2t}{2\sqrt{2}t} = \dfrac{1}{2}$ ∴

$\kappa(t) =$ _____

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: _____