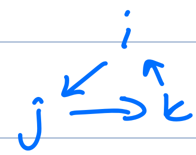
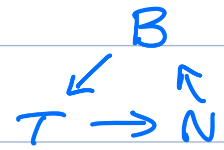
Calc 3 Exam 1 Review

- Things to know that are <u>not</u> on formula sheet

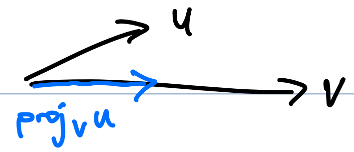
$i = \langle 1,0,0 \rangle \quad j = \langle 0,1,0 \rangle \quad k = \langle 0,0,1 \rangle$



cross product $|u \times v| = |u||v| \sin\theta$



$\text{proj}_v u = \dfrac{u \cdot v}{|v|^2} \vec{v}$



properties of vectors, dot products, & cross products

ex: $u \cdot v = v \cdot u$, but $u \times v \neq v \times u$

(see lectures 2-4) L7 p. 8

Distance formula: $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$ Also L5 p. 9

Equation of a line: $\langle$ starting point $\rangle$ + t * $\langle$ directional vector $\rangle$

Work = Force $\overset{\text{dot}}{\downarrow}\!\cdot$ Displacement dot product of F · Displacement

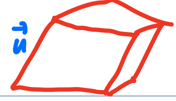
$W = |F||D|\cos\theta \qquad u \cdot v = |u||v|\cos\theta \qquad \langle F \rangle \cdot \langle D \rangle$

2D Shapes (for traces) 3D Shapes (for surfaces)

Arc length: $\displaystyle\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt \qquad a \le t \le b$

$\displaystyle\int_a^b |r'(t)|$

Area of parallelogram: $|u \times v|$ Area of triangle: $\dfrac{|u \times v|}{2}$

3D parallelogram: $\vec{u} \cdot (\vec{v} \times \vec{w})$ 

Speed $= |r'(t)| = |v(t)|$ Curvature: $\kappa(x) = \dfrac{|f''(x)|}{(1 + f'(x)^2)^{3/2}}$

Many people asked long-winded problems. Can post solutions, but wanted to focus on realistic

Practice Problems:

X9 #8 Find a_T & a_N

$r(t) = \langle \cos(3t), \sin(3t), 1 \rangle$ @ $t = \pi/4$

$v(t) = r'(t) = \langle -3\sin(3t), 3\cos(3t), 0 \rangle$

$a_T = \dfrac{v \cdot a}{|v|}$

$|v(t)| = \sqrt{9\sin^2(3t) + 9\cos^2(3t)}$

$|v(t)| = \sqrt{9\,[\cancel{\sin^2(3t) + \cos^2(3t)}]}$ 1

$|v(t)| = \sqrt{9} = 3$

$a(t) = r''(t) = \langle -9\cos(3t), -9\sin(3t), 0 \rangle$

$\dfrac{v \cdot a}{|v|} = \langle -3\sin(3t), 3\cos(3t), 0 \rangle \cdot \langle -9\cos(3t), -9\sin(3t), 0 \rangle$

$= \dfrac{27\sin(3t)\cos(3t) - 27\sin(3t)\cos(3t)}{3} = \dfrac{0}{3} = a_T$

$a_N = \dfrac{|v \times a|}{|v|} = \dfrac{27}{3} = 9$

$$\begin{matrix} i & j & k & i & j \\ -3s & 3c & 0 & -3s & 3c \\ -9c & -9s & 0 & -9c & -9s \end{matrix}$$

$\sin(3t) = s$
$\cos(3t) = c$

$0i + 0j + 27s^2 k - 0j - 0i + 27c^2 k$

$v \times a = \langle 0, 0, 27s^2 + 27c^2 \rangle$

$27\sin^2(3t) + 27\cos^2(3t) = 27$

$|v \times a| = \sqrt{27^2} = 27$

Fall 2019 FRQ

#1

see other document

Fall 2019 #2

$r'(t) = \langle -2t\sin(t^2),\ 2t\cos(t^2),\ 2t\rangle$

See other document

Some problem from some practice exam

<u>Equidistant</u> <u>(0,0,1)</u> & <u>plane z = -1</u>

distance

(x,y,z) to $(0,0,1)$ $\qquad$ (x,y,z) to $z=-1$ $\quad z=5$

$|z-(-1)| \quad |z-5|$

$$\left(\sqrt{(x-0)^2+(y-0)^2+(z-1)^2}\right)^2 = \left(|z+1|\right)^2$$

$$x^2+y^2+(z-1)^2 = (z+1)^2$$

$$x^2+y^2+\cancel{z^2}-2z\cancel{+1} = \cancel{z^2}+2z\cancel{+1}$$

$+2z \qquad +2z$

$$x^2+y^2=4z$$

$$\frac{x^2}{4}+\frac{y^2}{4}=z$$

circular paraboloid

$\langle t^2,\ 2t\sin(t),\ 2t\cos(t) \rangle$