

#1

$$\begin{aligned} \text{Step ①: Find } u - 2w &= \langle -2, -1, 3 \rangle - 2\langle 1, 2, -1 \rangle \\ &= \langle -2 - 2, -1 - 4, 3 - (-2) \rangle \\ &= \langle -4, -5, 5 \rangle = v. \end{aligned}$$

Step ②: Make this into a unit vector. Without this step, it's extremely tedious to make the vector have magnitude 5. Since unit vectors have magnitude 1, this allows us to just multiply it by 5 after (because $c|v| = |cv|$.)

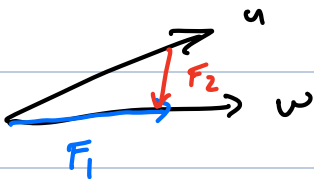
$$\hat{v} = \frac{v}{|v|} = \frac{\langle -4, -5, 5 \rangle}{\sqrt{16 + 25 + 25}} = \frac{1}{\sqrt{66}} \langle -4, -5, 5 \rangle$$

step ③: We want this in the opposite direction, so multiply by -1.

$$\Rightarrow v = -\frac{1}{\sqrt{66}} \langle -4, -5, 5 \rangle$$

step ④: Multiply by 5 for magnitude 5: $\boxed{-\frac{5}{\sqrt{66}} \langle -4, -5, 5 \rangle}$

#2



$$\begin{aligned} \text{proj}_w u &= \frac{u \cdot w}{|w|^2} w = \frac{(-2)(1) + (-1)(2) + (3)(-1)}{(\sqrt{1^2 + 2^2 + (-1)^2})^2} \langle 1, 2, -1 \rangle \\ &= \boxed{\frac{-7}{6} \langle 1, 2, -1 \rangle = F_1.} \end{aligned}$$

$$u = F_1 + F_2 \Rightarrow F_2 = u - F_1 = \boxed{\langle -2, -1, 3 \rangle - \left(-\frac{7}{6}\right) \langle 1, 2, -1 \rangle}$$

