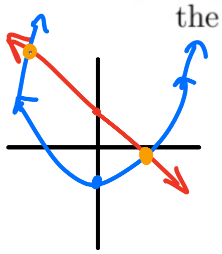
MAC2313 Quiz 5B

Name and section: Solution

1). (7 points) Calculate the integral of the function $f(x,y) = x + y$ inside the region bounded by the parabola $y = x^2 - 3$ and the line $y = -(1/2)x + 2$.



POI's: $x^2-3=-\frac{1}{2}x+2$

$\Rightarrow 2x^2-6=-x+4$

$\Rightarrow 2x^2+x-10=0$

$(2x+5)(x-2)=0$

$x=-5/2,\ 2$

set up ↑

$$\int_{-5/2}^{2}\int_{x^2-3}^{-\frac{1}{2}x+2}(x+y)\,dy\,dx = \int_{-5/2}^{2} xy+\frac{y^2}{2}\Big|_{x^2-3}^{-\frac{1}{2}x+2}\,dx$$

$$=\int_{-5/2}^{2} x\left(-\tfrac{1}{2}x+2\right)+\frac{\left(-\frac{1}{2}x+2\right)^2}{2}-\left[x(x^2-3)+\frac{(x^2-3)^2}{2}\right]dx$$

$$=\int_{-5/2}^{2} -\frac{1}{2}x^2+2x+\frac{1}{2}\left(\frac{1}{4}x^2-2x+4\right)-x^3+3x\ \frac{-x^4+6x^2-9}{2}\,dx$$

$$=-\frac{1}{6}x^3+x^2+\frac{1}{2}\left(\frac{1}{12}x^3-x^2+4x\right)-\frac{x^4}{4}+\frac{3}{2}x^2+\frac{1}{2}\left(-\frac{x^5}{5}+2x^3-9x\right)\Big|_{-5/2}^{2}$$

2). (3 points) Convert the following from spherical $(\overset{\rho}{\cancel{r}},\theta,\phi)$ to rectangular (x,y,z) coordinates:

$$(5,\frac{\pi}{4},\frac{\pi}{2})$$

$x=\rho\sin\varphi\cos\theta \qquad y=\rho\sin\varphi\sin\theta \qquad z=\rho\cos\varphi$

$x=5\sin(\pi/2)\cos(\pi/4) \qquad y=5\sin(\pi/2)\sin(\pi/4) \qquad z=5\cos(\pi/2)$

$x=5(1)(\sqrt{2}/2) \qquad y=5(1)(\sqrt{2}/2) \qquad z=5(0)$

$\Rightarrow \left(5\sqrt{2}/2,\ 5\sqrt{2}/2,\ 0\right)$