#1 Easy way:

$$\int 4x^3 z\,dx = \underline{x^4 z}$$

$$\int z^2 e^{4y}\,dy = \frac{z^2 e^{4y}}{\underline{4}}$$

$$\int \left(x^4 + \tfrac{1}{2} z e^{4y} - 3z\right) dz = \underbrace{x^4 z}_{\text{duplicate}} + \underbrace{\frac{z^2 e^{4y}}{4}}_{\text{duplicate}} - \frac{3}{2} z^2$$

$$\Rightarrow \quad f(x,y,z) = x^4 z + \frac{z^2 e^{4y}}{4} - \frac{3}{2} z^2 + C$$

#1 Long way:

$$f(x,y,z) = \int 4x^3 z\,dx = x^4 z + g(y,z)$$

$$f_y(x,y,z) = \int g_y(y,z) = \int z^2 e^{4y}\,dy$$

$$\Rightarrow \quad g(y,z) = \frac{z^2 e^{4y}}{4} + h(z)$$

w.r.t. z

$$f_z(x,y,z) = x^4 + \underline{g_z(y,z)} = x^4 + \tfrac{1}{2} z e^{4y} - 3z$$

$$\Rightarrow \cancel{x^4} + \cancel{\frac{z e^{4y}}{2}} + h_z(z) = \cancel{x^4} + \cancel{\frac{1}{2} z e^{4y}} - 3z$$

$$\Rightarrow \int h_z(z) = \int -3z \, dz$$

$$h(z) = \frac{-3z^2}{2} + c$$

$$\Rightarrow f_z(x,y,z) = x^4 z + \frac{z^2 e^{4y}}{4} - \frac{3}{2} z^2 + c$$

#2 since C is a circle of radius 3 oriented counterclockwise, we let $x = 3\cos(t)$ and $y = 3\sin(t)$.

Then $r(t) = \langle 3\cos(t), 3\sin(t) \rangle$, so

$$r'(t) = \langle -3\sin(t), 3\cos(t) \rangle.$$

Therefore, $\int_C F \cdot dr = \int_C \langle 9\cos(t)\sin(t), -9\cos^2(t) \rangle \cdot \langle -3\sin(t), 3\cos t \rangle dt$

$$= \int_C -27\cos(t)\sin^2(t) - 27\cos^3(t) \, dt.$$

Since we only want the upper half of C, we have:

$$-27 \int_0^{\pi} \cos(t) \left[\sin^2(t) + \cos^2(t) \right] dt$$

$$= -27 \int_0^{\pi} \cos(t)\,dt = -27 \left[\sin(t)\right]_0^{\pi} = 0.$$