University of Florida **MAA 4102/5104**

Homework #6 – **Continuity of a function, properties of continuous functions**

Exercise 1. Determine where the given functions are continuous. Explain clearly. (First, find the domain of the given functions.)

1. $f(x) = |x-1|$
2. $f(x) = \frac{|x|}{x}$
3. $$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$
4. $$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$
5. $f(x) = \lfloor x \rfloor$
6. $f(x) = \frac{1}{\sqrt{x}}$

Exercise 2. Prove that if the functions $f, g \colon D \to \mathbb{R}$ are continuous at a, then the given functions are also continuous at a.

1. $|f|$
2. $\min(f, g)$
3. $\max(f, g)$

Exercise 3. Prove of find a counterexample to the following statements.

1. f bounded on $[a, b]$ implies that f is continuous on $[a, b]$.
2. f continuous on (a, b) implies that f is bounded on (a, b).
3. f^2 continuous on $[a, b]$ implies that f is continuous on $[a, b]$.
4. $f + g$ and f continuous on (a, b) implies that g is continuous on (a, b).
5. fg and f continuous on (a, b) implies that g is continuous on (a, b).
6. $|f|$ continuous on $[a, b]$ implies that f is continuous on $[a, b]$.
7. f is continuous at c if and only if for any sequence $\{x_n\}$ converging to c, the sequence $\{f(x_n)\}$ converges to $f(c)$.

Exercise 4. Give examples of the following requested functions, if possible.

1. function f defined on $\mathbb{R}$ but not continuous at any point of $\mathbb{R}$.
2. function f defined on $\mathbb{R}$ and continuous at exactly one point of $\mathbb{R}$.

Exercise 5. Give an example of a function $f\colon [a,b] \to \mathbb{R}$ that is not continuous but whose range is

1. an open and bounded interval.
2. an open and unbounded interval.
3. a closed and unbounded interval.