MAC 2313 Exam I, Part II Free Response

Name: Key Discussion Period ________

Circle your TA's Name

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SHOW ALL WORK TO RECEIVE FULL CREDIT

1. Consider points $P(0,2,1)$, $Q(3,0,1)$ and $R(3,2,0)$ and vector $\vec{u} = \langle 1, -3, 2\rangle$.

   (a) (5 points) Find two unit vectors parallel to $\overrightarrow{PQ} + \vec{u}$

   $\overrightarrow{PQ} = \langle 3-0, 0-2, 1-1 \rangle = \langle 3, -2, 0 \rangle$

   $\overrightarrow{PQ} + \vec{u} = \langle 3+1, -2-3, 0+2 \rangle = \langle 4, -5, 2 \rangle$

   $|\overrightarrow{PQ} + \vec{u}| = \sqrt{(4)^2 + (-5)^2 + (2)^2} = \sqrt{16+25+4} = \sqrt{45}$

   $v_1 = \frac{1}{\sqrt{45}}\langle 4, -5, 2\rangle = \left\langle \frac{4}{\sqrt{45}}, \frac{-5}{\sqrt{45}}, \frac{2}{\sqrt{45}} \right\rangle$

   $v_2 = \frac{-1}{\sqrt{45}}\langle 4, -5, 2\rangle = \left\langle \frac{-4}{\sqrt{45}}, \frac{5}{\sqrt{45}}, \frac{-2}{\sqrt{45}} \right\rangle$

   (b) (5 points) Find the area of the triangle $\triangle PQR$.

   $\overrightarrow{PQ} = \langle 3, -2, 0 \rangle$ from (a)

   $\overrightarrow{PR} = \langle 3-0, 2-2, 0-1 \rangle = \langle 3, 0, -1 \rangle$

   $$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 3 & -2 & 0 \\ 3 & 0 & -1 \end{vmatrix} = i(2-0) - j(-3-0) + k(0+6) = \langle 2, 3, 6 \rangle$$

   $|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{4+9+36} = \sqrt{49} = 7$

   $A = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{7}{2}$

2. (6 points) The position function of a particle is given by $\vec{r}(t) = \langle t^2, 3t, t^2 - 8t\rangle$. When is the speed a minimum?

$v(t) = r'(t) = \langle 2t,\ 3,\ 2t-8 \rangle$

speed $= |v(t)| = \sqrt{(2t)^2 + (3)^2 + (2t-8)^2} = \sqrt{4t^2 + 9 + 4t^2 - 32t + 64}$

$= \sqrt{8t^2 - 32t + 73}$

$|v(t)|' = \frac{1}{2}(8t^2 - 32t + 73)^{-1/2}(16t - 32)$

$0 = \frac{1}{2}(8t^2 - 32t + 73)^{-1/2}(16t - 32)$

$0 \neq \dfrac{1}{2\sqrt{8t^2 - 32t + 73}}$ $\qquad 0 = 16t - 32$

$32 = 16t \implies t = 2$

3. (6 points) Find the equation of the plane parallel to the vectors $\langle -1, 0, 1\rangle$ and $\langle 1, -1, 0\rangle$ containing the point $(1, 2, 3)$

$\vec{u}$ $\qquad \vec{v}$

$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = i(0+1) - j(0-1) + k(1-0) = \langle 1, 1, 1\rangle$

$x + y + z = d$

$(1) + (2) + (3) = d$

$6 = d$

$x + y + z = 6$

OR

$\langle 1, 1, 1\rangle \cdot \langle x-1,\ y-2,\ z-3\rangle = 0$

$x - 1 + y - 2 + z - 3 = 0$

$x + y + z - 6 = 0$

4. (6 points) Find the curvature of $y = 4x^3$ at $x = 1$.

$\kappa = \dfrac{|f''|}{(1 + (f')^2)^{3/2}}$

$f' = 12x^2$

$f'' = 24x$

$|f''| = \sqrt{(24x)^2} = 24x$

$\kappa = \dfrac{24x}{(1 + 144x^4)^{3/2}}$

$\kappa(1) = \dfrac{24}{(145)^{3/2}}$

OR

$\kappa = \dfrac{|v \times a|}{|v|^3}$

$r(x) = \langle x, 4x^3, 0\rangle$

$v(x) = \langle 1, 12x^2, 0\rangle$

$a(x) = \langle 0, 24x, 0\rangle$

$(v \times a)(1) \begin{vmatrix} i & j & k \\ 1 & 12 & 0 \\ 0 & 24 & 0 \end{vmatrix} = i(0-0) - j(0-0) + k(24-0)$

$= \langle 0, 0, 24\rangle$

$|v \times a|(1) = \sqrt{(24)^2} = 24$

$|v|(1) = \sqrt{(1)^2 + (12)^2 + 0^2} = \sqrt{145}$

$\kappa(1) = \dfrac{24}{(\sqrt{145})^3} = \dfrac{24}{(145)^{3/2}}$