## MAC 2313 Exam I, Part II Free Response

Name: Key Discussion Period ________

Circle your TA's Name

| | | | |
|---|---|---|---|
| Carl Ye | Deep Kundu | Christian Austin | Michael Coopman |
| Vincent Holmlund | Aditya DeSaha | Abby Owens | Emmanuel Okon |
| Erik Shute | Joshua Arroyo | Michael Waite | |
| Jiun Cho | Satyanath Howladar | Vangmay Jayant | |

SHOW ALL WORK TO RECEIVE FULL CREDIT

**1.** (12 points) Let $S$ be the surface consisting of all the points $P(x, y, z)$ for which the distance from $P$ to the $x$-axis is two times the distance from $P$ to the $yz$-plane.

(a) Find an equation of $S$. (Express your answer without radicals)

distance from $(x,y,z)$ to x axis: $\sqrt{y^2+z^2} = D$

distance from $(x,y,z)$ to $yz$-plane: $|x| = d$

$D = 2d$

$\sqrt{y^2+z^2} = 2|x|$

$y^2+z^2 = 4x^2$

$y^2+z^2=4x^2$
(or $-4x^2+y^2+z^2=0$).

(b) Discuss traces and identify the surface $S$.

- Traces parallel to the $xy$-plane ($z = k$) are lines if $k = 0$ $\left\{\begin{array}{l} y^2 = 4x^2 \\ y = \pm 2x \end{array}\right.$

  $-4x^2+y^2 = -k^2$ and hyperbolas if $k \neq 0$

- Traces parallel to the $yz$-plane ($x = k$) are a point if $k = 0$ $\left\{\begin{array}{l} y^2+z^2=0 \\ (y=0=z) \end{array}\right.$

  $y^2+z^2 = 4k^2$ and circles if $k \neq 0$

- Traces parallel to the $xz$-plane ($y = k$) are lines if $k = 0$ $\left\{\begin{array}{l} z^2=4x^2 \\ z=\pm 2x \end{array}\right.$

  $-4x^2+z^2 = -k^2$ and hyperbolas if $k \neq 0$

Therefore, $S$ is a cone.

**2.** (16 points) Let $C$ be a smooth curve parameterized by $\vec{r}(t) = \langle \cos(2t^2), \sin(2t^2), t^2 \rangle$ for $t \geq 0$, find (simplify your final answers including radicals)

(a) $|\vec{r}'(t)| =$ $2\sqrt{5}\,t$

$$r'(t) = \langle -4t\sin(2t^2),\ 4t\cos(2t^2),\ 2t \rangle$$

$$|r'(t)| = \sqrt{16t^2\sin^2(2t^2) + 16t^2\cos^2(2t^2) + 4t^2}$$

$$= \sqrt{16t^2 + 4t^2} = \sqrt{20t^2} = 2\sqrt{5}\,t$$

(b) the arc length of $C$ from $t = 0$ to $t = 2$

$$L = \int_0^2 2\sqrt{5}\,t\,dt$$

$$= \sqrt{5}\,t^2 \Big|_0^2$$

$$= 4\sqrt{5}$$

$4\sqrt{5}$

(c) the curvature of the curve $C$ for any $t \neq 0$

$$\kappa = \frac{|\hat{T}'(t)|}{|r'(t)|} = \left(\frac{8t}{\sqrt{5}}\right)\left(\frac{1}{2\sqrt{5}\,t}\right) = \frac{4}{5}$$

$$\hat{T}(t) = \frac{r'(t)}{|r'(t)|} = \frac{1}{2\sqrt{5}\,t}\langle -4t\sin(2t^2),\ 4t\cos(2t^2),\ 2t \rangle$$

$$= \left\langle \frac{-2}{\sqrt{5}}\sin(2t^2),\ \frac{2}{\sqrt{5}}\cos(2t^2),\ \frac{1}{\sqrt{5}} \right\rangle$$

$$\hat{T}'(t) = \left\langle \frac{-8t}{\sqrt{5}}\cos(2t^2),\ \frac{-8t}{\sqrt{5}}\sin(2t^2),\ 0 \right\rangle$$

$$|\hat{T}'(t)| = \sqrt{\frac{64t^2}{5}\cos^2(2t^2) + \frac{64t^2}{5}\sin^2(2t^2)}$$

$$= \sqrt{\frac{64t^2}{5}} = \frac{8t}{\sqrt{5}}$$

$\kappa(t) =$ $\frac{4}{5}$

**University of Florida Honor Pledge:**

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ____________________