MAC 2313 Exam I, Part II Free Response

Name: _____ Discussion Period _____

Circle your TA's Name

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SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (6 points) Find the parametric equations for the line L passing through the point (3,7,12) and perpendicular to the plane $x - y + 3z = 1$.

$\vec{v} = \langle 1, -1, 3 \rangle$

$\vec{r}(t) = \langle 3,7,12 \rangle + t\langle 1,-1,3 \rangle$

$x = 3 + t$

$y = 7 - t$

$z = 12 + 3t$

2. (8 points) Find the parametric equations for the line of intersection of the planes $3x + 2y - 5z = 1$ and $3x - 2y - z = 0$.

Let $x = 0$ $\quad 2y - 5z = 1$

$-2y - z = 1$

$-6z = 1$

$z = -\frac{1}{6}$

$2y + 5\left(\frac{1}{6}\right) = 1$

$2y = 1 - \frac{5}{6}$

$2y = \frac{1}{6}$

$y = \frac{1}{12}$

any point on line

$\left(0, \frac{1}{12}, -\frac{1}{6}\right)$

$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -5 \\ 3 & -2 & -1 \end{vmatrix} = \langle -12, -12, -12 \rangle = \langle -1, -1, -1 \rangle$

or $\langle 1, 1, 1 \rangle$

$x = -12t$

$y = \frac{1}{12} - 12t$

$z = -\frac{1}{6} - 12t$

or

$x = -t$

$y = \frac{1}{12} - t$

$z = -\frac{1}{6} - t$

3. (6 points) Determine whether the lines

$$L_1 : x = 2 + t \qquad y = 21 + 7t, \qquad z = 15 + 4t$$

$$L_2 : x = -5 + 2s, \qquad y = -18 + 9s, \qquad z = -11 + 7s$$

intersect, are skew, or are parallel. If they intersect determine the point of intersection; if not leave that part of the question blank.

Circle One: Intersect or Skew or Parallel

$2+t=-5+2s \quad 21+7t=-18+9s \quad 15+4t=-11+7s$

$t=-7+2s \quad 21+7(-7+2s)=-18+9s$

$21-49+14s=-18+9s$

$-10=-5s$

$s=2$

then

$t=-7+2(2)$

$t=-3$

ck $15+4(-3)=11+7(2)$

$3=3$ ✓

$x=2-3=-1$

$y=21+7(-3)=0$

$z=15+4(-3)=3$

Point of Intersection $(-1,0,3)$

4. (8 points) Decompose the vector $\vec{u} = \langle 3, -13\rangle$ into two orthogonal vectors, one parallel to $\vec{v} = \langle 1, 2\rangle$ and one perpendicular to $\vec{v}$.

$$\text{proj}_{\vec{v}}\vec{u} = \frac{\langle 3,-13\rangle \cdot \langle 1,2\rangle}{1+4}\langle 1,2\rangle$$

$$= \frac{3-26}{5}\langle 1,2\rangle$$

$$= -\frac{23}{5}\langle 1,2\rangle$$

$$\vec{v}_{\parallel} = \left\langle -\frac{23}{5}, -\frac{46}{5}\right\rangle$$

$\vec{u} = \vec{v}_{\parallel} + \vec{v}_{\perp}$

$\vec{v}_{\perp} = \vec{u} - \vec{v}_{\parallel}$

$= \langle 3,-13\rangle - \left\langle -\frac{23}{5}, -\frac{46}{5}\right\rangle$

$$\vec{v}_{\perp} = \left\langle \frac{38}{5}, -\frac{19}{5}\right\rangle$$