MAC 2313 Exam II, Part II Free Response

Name: Key Discussion Period ________

Circle your TA's Name

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SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (12 points) Use Lagrange Multipliers to find extreme values of the function $f(x, y) = 3x - 2y$ subject to the constraint $x^2 + y^2 = 44$. $g(x,y) = x^2 + y^2 - 44$

$\nabla f = \lambda \nabla g$

$\langle 3, -2 \rangle = \lambda \langle 2x, 2y \rangle$

$3 = \lambda 2x \qquad -2 = \lambda 2y$

$\frac{3}{2x} = \lambda \qquad -1 = \lambda y$

$\frac{-1}{y} = \lambda$

$\frac{3}{2x} = \frac{-1}{y}$

$-2x = 3y$

$\frac{-2x}{3} = y$

$x^2 + \left(\frac{-2x}{3}\right)^2 - 44 = 0$

$x^2 + \frac{4x^2}{9} = 44$

$9x^2 + 4x^2 = 396$

$13x^2 = 396$

$x = \pm\sqrt{\frac{396}{13}}$

$\left.\begin{array}{l} \frac{-2\left(\sqrt{\frac{396}{13}}\right)}{3} = y \\ \frac{-2\sqrt{396}}{3\sqrt{13}} = y \end{array}\right\} p_1 = \left(\sqrt{\frac{396}{13}}, \frac{-2\sqrt{396}}{3\sqrt{13}}\right)$

$\left.\begin{array}{l} \frac{-2\left(-\sqrt{\frac{396}{13}}\right)}{3} = y \\ \frac{2\sqrt{396}}{3\sqrt{13}} = y \end{array}\right\} p_2 = \left(-\sqrt{\frac{396}{13}}, \frac{2\sqrt{396}}{3\sqrt{13}}\right)$

$f(p_1) = \frac{3\sqrt{396}}{\sqrt{13}} + \frac{4\sqrt{396}}{3\sqrt{13}} = \frac{13\sqrt{396}}{3\sqrt{13}} \leftarrow \text{max}$

$f(p_2) = \frac{-3\sqrt{396}}{\sqrt{13}} - \frac{4\sqrt{396}}{3\sqrt{13}} = \frac{-13\sqrt{396}}{3\sqrt{13}} \leftarrow \text{min}$

2. Let $f(x, y) = x - y^2$.

(a) (6 points) Find an equation of the tangent plane to the surface $z = f(x, y)$ at $(1, 2)$.

$f_x = 1$ $\quad$ $f_y = -2y$ $\quad$ $f(1,2) = 1-4 = -3$

$f_x(1,2) = 1$ $\quad$ $f_y(1,2) = -4$

$z = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$z = -3 + (x-1) - 4(y-2)$

$z = x - 4y + 4$

$z =$ $x-4y+4$

(b)(5 points) Find the maximum rate of increase/decrease of f at $(1, 2)$, and indicate the direction where it occurs.

$\nabla f = \langle 1, -2y \rangle$

$\nabla f(1,2) = \langle 1, -4 \rangle$

$|\langle 1, -4 \rangle| = \sqrt{1+16} = \sqrt{17}$

max rate of increase = $\sqrt{17}$ in the direction $\langle 1, -4 \rangle$

max rate of decrease = $-\sqrt{17}$ in the direction $\langle -1, 4 \rangle$

(c) (5 points) Is there a unit vector $\hat{u}$ so that the rate of change of f at $(1, 2)$ in the direction $\hat{u}$ is 3?

$D = \nabla f \cdot u$

$3 = \langle 1, -4 \rangle \cdot \langle x, y \rangle$

$$\left.\begin{aligned} x^2 + y^2 &= 1 \\ 3 &= x - 4y \end{aligned}\right\}$$ can find $\vec{u}$ by solving this system, but we know it will exist by reason below

Yes or No (circle one), because $-\sqrt{17} < 3 < \sqrt{17}$