MAC 2313 Exam II, Part II Free Response

Name: ________________________________ Discussion Period ________

Circle your TA's Name

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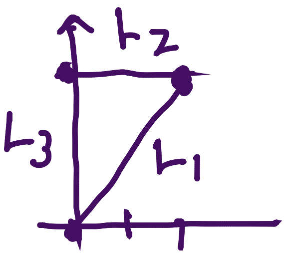
SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (8 points) Find the absolute maximum and minimum values of $f(x, y) = 2x^2 - 8x + y^2 - 8y + 2$ on the closed triangle with vertices (0,0), $(0, 4)$, and $(2, 4)$.



Interior

$f_x = 4x - 8 = 0 \quad x = 2 \quad (2,4)$ on the boundary

$f_y = 2y - 8 = 0 \quad y = 4$

Boundary

L1 $\quad y = 2x \quad (0,0) \ (2,4)$

$f(x,2x) = 2x^2 - 8x + (2x)^2 - 8(2x) + 2$

$= 2x^2 - 8x + 4x^2 - 16x + 2$

$= 6x^2 - 24x + 2$

$f'(x,2x) = 12x - 24 = 0 \quad x = 2 \quad y = 4$

L2 $\quad y = 4 \quad 0 \le x \le 4$

$f(x,4) = 2x^2 - 8x + 16 - 32 + 2$

$f(x,4) = 2x^2 - 8x - 14$

$f'(x,4) = 4x - 8 \quad x = 2 \quad (2,4)$

L3 $\quad x = 0 \quad f(0,y) = y^2 - 8y + 2$

$f'(0,y) = 2y - 8 \quad y = 4 \quad (0,4)$

max

$f(0,0) = 2$

$f(0,4) = -14$

$f(2,4) = -22$

min

2. Consider the function $f(x, y) = x^2 - 5xy$.

(a) (4 points) Find $\nabla f(x, y)$.

$\nabla f(x,y) = \langle 2x-5y, -5x \rangle$

(b) (4 points) Find the directional derivative at $(2, 1)$ in the direction of $\vec{v} = -\hat{\imath} + 3\hat{\jmath}$.

$|\vec{v}| = \sqrt{1+9} = \sqrt{10}$ $\hat{v} = \langle -\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \rangle$

$\nabla f(2,1) = \langle -1, -10 \rangle$

$D_{\hat{v}} f(2,1) = \langle -1, -10 \rangle \cdot \langle -\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \rangle = +\frac{1}{\sqrt{10}} - \frac{30}{\sqrt{10}} = -\frac{29}{\sqrt{10}}$

(c) (4 points) What is the value of the functions maximum rate of change at (2,1)?

$|\nabla f(2,1)| = \sqrt{1+10^2} = \sqrt{101}$

(d) (4 points) Find the linearization of f at (2,1).

$f(2,1) = 4 - 5(2)(1)$
$= -6$

$z = f(2,1) - (x-2) - 10(y-1)$

$L(x,y) = -6 - (x-2) - 10(y-1)$

(e) (4 points) Use the linearization to approximate $f(1.9, 0.9)$.

$L(1.9, 0.9) = -6 - (1.9-2) - 10(0.9-1)$

$= -6 + .1 + 1$

$= -4.9$