MAC 2313 Exam II, Part II Free Response

Name: Key Discussion Period ________

Circle your TA's Name

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SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (14 points) Let $f(x, y) = x - y^2$.

(a) Find an equation of the tangent plane to the surface $z = f(x, y)$ at $(1, 2)$.

$\nabla f = \langle 1, -2y \rangle$

$\nabla f(1,2) = \langle 1, -4 \rangle$

$\langle 1, -4 \rangle \cdot \langle x-1, y-2 \rangle + f(1,2) = z$

$x - 1 - 4(y-2) - 3 = z$

$z =$ $x - 4y + 4$

(b) Find the maximum rate of increase/decrease of f at $(1, 2)$, and indicate the direction where it occurs.

$|\nabla f(1,2)| = \sqrt{(1)^2 + (-4)^2} = \sqrt{17}$

max rate of increase = $\sqrt{17}$ in the direction $\langle 1, -4 \rangle$

max rate of decrease = $-\sqrt{17}$ in the direction $\langle -1, 4 \rangle$

(c) Is there a unit vector $\hat{u}$ so that the rate of change of f at $(1, 2)$ in the direction $\hat{u}$ is 3? State your reason.

Yes or No (circle one), because $-\sqrt{17} \le (D_{\hat{u}} f(1,2) = 3) \le \sqrt{17}$

2. (14 points) Find the point in the first quadrant on the hyperbola $xy = 36$ where the value $1 - 4x - y$ is a maximum.

(a) Set up the optimization problem as

Maximize $f(x, y) =$ $1-4x-y$

Subject to the constraint $xy = 36$

(b) Solve the optimization problem using the Method of Lagrange Multipliers.

Note: You do not need to show the extreme value is a maximum.

$f = 1-4x-y \qquad g = xy-36$

$\nabla f = \langle -4, -1 \rangle \qquad \nabla g = \langle y, x \rangle$

$\nabla f = \lambda \nabla g$

$\langle -4, -1 \rangle = \lambda \langle y, x \rangle$

$-4 = \lambda y \qquad -1 = \lambda x$

$\frac{-4}{y} = \lambda \qquad \frac{-1}{x} = \lambda$

$\frac{-4}{y} = \frac{-1}{x}$

$4x = y$

plug into constraint

$36 = x(4x)$

$9 = x^2$

$\pm 3 = x$

* first quadrent $\Rightarrow +3$ only

$4(3) = y$

$12 = y$

$f(3, 12) = 1 - 4(3) - 12$

$= -23$

The maximum value is -23 and it occurs at the point $(3, 12)$

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ____________________