Name: Key	Discussion Period		
	Circle your TA's Name		
Carl Ye	Xiaochen Duan	Aritram Dhar	Michelle Baker
Kushagri Sharma	Arya Memana	Abby Owens	Saketh Narayanan
Umesha Wijerathne	Jacob Levenson	Eros Sunny	Shi Zhou

MAC 2313 Exam II, Part II Free Response

SHOW ALL WORK TO RECEIVE FULL CREDIT

- 1. (14 points) Let E be the solid enclosed by $z = \sqrt{3x^2 + 3y^2}$ and z = 3 in the <u>first octant</u>.
 - (a) Write the integral in cylindrical coordinates. (Show your work for the upper bound of r.) $z = \sqrt{3x^2 + 3y^2}$ z = 3



$$\iiint_E \frac{1}{\sqrt{x^2 + y^2 + z^2}} \, dV = \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{3}} \int_{\sqrt{3}}^{3} \frac{r}{\sqrt{r^2 + z^2}} \, dz \, dr \, d\theta$$

(b) Write the integral in spherical coordinates. (Show your work for the upper bound of ρ and ϕ .)

$$z = \sqrt{3x^{2} + 3y^{2}}$$

$$z = 3$$

$$p \cos \phi = \sqrt{3} p \sin \phi$$

$$p \cos \phi = 3$$

$$p = 3 \sec \phi$$

$$p = 3 \sec \phi$$

$$p = 3 \sec \phi$$

$$\frac{\sin \phi}{\sin \phi} = \sqrt{3}$$

$$\int \int \int_{E} \frac{1}{\sqrt{x^{2} + y^{2} + z^{2}}} dV = \int_{0}^{\pi} \frac{7}{2} \int_{0}^{\pi} \frac{p^{2} \sin \phi}{\rho} d\rho d\phi d\theta$$

2. (14 points) Let R be the region enclosed by the ellipse $\frac{x^2}{16} + y^2 = 1$ in the <u>first quadrant</u>. Use appropriate change of variables to evaluate

$$\iint_{R} y^{2} dA$$

$$X = 4u \quad y = V \qquad J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = 4$$

$$\int_{0}^{\frac{\pi}{2}} \int_{0}^{1} 4r^{3} \sin^{2} \theta \, dr \, d\theta = \int_{0}^{\frac{\pi}{2}} r^{4} \sin^{2} \theta \Big|_{0}^{1} \, d\theta$$

$$= \int_{0}^{\frac{\pi}{2}} \sin^{2} \theta \, d\theta - \int_{0}^{\frac{\pi}{2}} \frac{1}{2} - \frac{\cos(2\theta)}{2} \, d\theta$$

$$= \frac{1}{2} \Theta - \frac{\sin(2\theta)}{4} \Big|_{0}^{\frac{\pi}{2}} = \frac{\pi}{4}$$

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid during this exam. Signature: