

MAC 2313 Exam II, Part II Free Response

Name: _____ Discussion Period _____

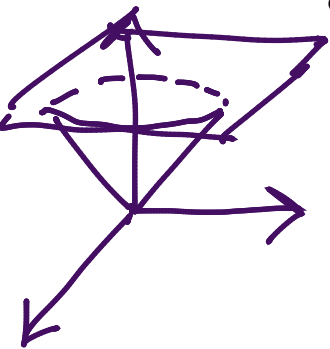
Circle your TA's Name

- | | | | |
|-------------------|---------------|------------------|----------------|
| Carl Ye | Kyle Adams | Christian Austin | Michelle Baker |
| Aditya DeSaha | Dylan Connell | Abby Owens | Julian Michele |
| Umesha Wijerathne | Chi Ding | David Maynoldi | Michaele Waite |
| Lezhi Liu | | | |

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (14 points) Let E be the solid enclosed by $z = \sqrt{3x^2 + 3y^2}$ and $z = 6$ in the first octant.

(a) Write the integral in cylindrical coordinates. (Show your work for the upper bound of r)



$$\begin{aligned}
 6 &= \sqrt{3x^2 + 3y^2} \\
 36 &= 3(x^2 + y^2) \\
 x^2 + y^2 &= 12 \\
 r &= 2\sqrt{3}
 \end{aligned}$$

$$\iiint_E \sqrt{x^2 + y^2 + z^2} dV = \int_0^{\frac{\pi}{2}} \int_0^{2\sqrt{3}} \int_{\sqrt{3}r}^6 r \sqrt{r^2 + z^2} dz dr d\theta$$

(b) Write the integral in spherical coordinates. (Show your work for the upper bound of ρ and ϕ)

$$\begin{aligned}
 z &= 6 \\
 \rho \cos \phi &= 6 \\
 \rho &= 6 \sec \phi
 \end{aligned}$$

$$\begin{aligned}
 z &= \sqrt{3} r \\
 \rho \cos \phi &= \sqrt{3} \rho \sin \phi \\
 \tan \phi &= \frac{1}{\sqrt{3}}
 \end{aligned}$$

$$\iiint_E \sqrt{x^2 + y^2 + z^2} dV = \int_0^{\frac{\pi}{2}} \int_0^{\pi/6} \int_0^{6 \sec \phi} \rho^3 \sin \phi d\rho d\phi d\theta$$

2. (14 points) Let R be the region enclosed by the ellipse $\frac{x^2}{4} + y^2 = 1$ and above the x -axis.
Use appropriate change of variables to evaluate

$$\iint_R x^2 dA$$

$$\begin{aligned} x &= 2u \\ y &= v \end{aligned}$$

$$J(u, v) = \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 2$$

$$\iint_R x^2 dA = \iint_S 2(2u)^2 dA$$

$$= 8 \iint_S u^2 dA$$

$$= 8 \int_0^\pi \int_0^1 r^3 \cos^2 \theta dr d\theta$$

$$= 4 \int_0^\pi (1 + \cos 2\theta) d\theta \int_0^1 r^3 dr$$

$$= 4 \left(\theta + \frac{1}{2} \sin 2\theta \right) \frac{r^4}{4}$$

$$\leq \pi$$

$$\iint_R x^2 dA = \underline{\pi}$$

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: _____