UF UNIVERSITY of FLORIDA

Department of Mathematics

MAC 2313
Final Exam B
Fall 2019

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) **SKIP** Section number

C. Under "special codes" code in the test ID numbers 4, 2.

1 2 3 ● 5 6 7 8 9 0
1 ● 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode B.

A ● C D E

E. 1) This test consists of 22 multiple choice questions. The test is counted out of 100 points, and there are 10 bonus points available.

2) The time allowed is 120 minutes.

3) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas within one day after the exam.

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ____________________________________

Summary of Integration Formulas

- Fundamental Theorem of Calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

- Fundamental Theorem of Line Integrals

$$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

- Green's Theorem (circulation form)

$$\iint_D \operatorname{curl} \vec{F} \cdot \hat{k}\,dA = \oint_C \vec{F} \cdot d\vec{r}$$

- Stokes' Theorem

$$\iint_S \operatorname{curl} \vec{F} \cdot \hat{n}\,dS = \oint_C \vec{F} \cdot d\vec{r}$$

- Green's Theorem (flux form)

$$\iint_D \operatorname{div} \vec{F}\,dA = \oint_C \vec{F} \cdot \hat{n}\,ds$$

- Divergence Theorem

$$\iiint_E \operatorname{div} \vec{F}\,dV = \oiint_S \vec{F} \cdot \hat{n}\,dS$$

NOTE: Be sure to bubble the answers to questions 1−22 on your scantron.

Questions 1 − 22 are worth 5 points each.

1. Let $\vec{F}(x, y, z) = \langle x, -2yz, 3xz^2 \rangle$. Which of the following vectors is orthogonal to curl $\vec{F}$ at the point $(1, 1, 1)$?

$$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -2yz & 3xz^2 \end{vmatrix} = \langle 2y, -3z^2, 0 \rangle$$

a. $\langle 2, -3, 0 \rangle$

b. $\langle 2, 3, 0 \rangle$

c. $\langle 3, -2, 5 \rangle$

d. $\langle -3, -2, 1 \rangle$

e. $\langle -3, 2, -1 \rangle$

@(1,1,1) gives $\langle 2, -3, 0 \rangle$

orthogonal means dot product is zero

$\langle 2, -3, 0 \rangle \cdot \langle -3, -2, 1 \rangle = -6 + 6 + 0 = 0$

* if you wanted parallel, check which is off by a scalar

2. Let $\vec{F} = \langle x^2 - y^2, -2xy + y \rangle$. Which of the following statements must be correct?

✗ P. $\nabla \cdot \vec{F} = 0$. $\langle \partial x, \partial y \rangle \cdot \langle x^2 - y^2, -2xy + y \rangle = 2x - 2x + 1 = 1$

✓ Q. $\vec{F}$ is conservative. $\int x^2 - y^2 \, dx$, $\int -2xy + y \, dy$ are solvable

✗ R. $\int_C \vec{F} \cdot d\vec{r} = 0$ for any ~~smooth curve~~ closed path C.

a. P and Q only

b. Q only

c. P and R only

d. Q and R only

e. P, Q, and R

3. Evaluate the line integral $\int_C 2xe^y\,ds$, where C is the line segment from $(0,0)$ to $(3,1)$.

a. 6

b. $6e$

c. $6(e-1)$

d. $6\sqrt{10}\,(e-1)$

e. $6\sqrt{10}$

$(0,0) \to (3,1) \Rightarrow r(t) = \langle 3t, t \rangle$ with $0 \le t \le 1$

$dr = \langle 3, 1 \rangle \quad |dr| = \sqrt{9+1} = \sqrt{10}$

$F = 2xe^y \quad F(t) = 2(3t)e^t = 6te^t$

$\int_C 2xe^y\,ds = \int_0^1 F \cdot |dr|\,dt = \int_0^1 \sqrt{10} \cdot 6te^t\,dt$

$t \quad + \quad e^t$
$1 \quad - \quad e^t$
$0 \quad \quad e^t$

$= 6\sqrt{10}\int_0^1 te^t\,dt = 6\sqrt{10}(te^t - e^t)\Big|_0^1$

$= 6\sqrt{10}(e-e) - 6\sqrt{10}(0-1) = 6\sqrt{10}$

$* \; e^0 = 1$

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4. Calculate $\oint_C \frac{y}{2}\,dx$, where C is the counterclockwise oriented curve bounding the triangle with vertices $(0,0)$, $(4,0)$, and $(1,3)$.

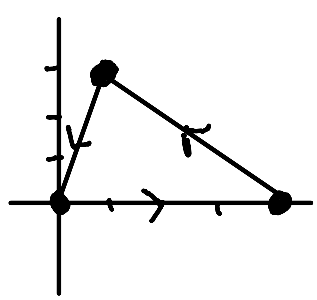
a. -3

b. -6

c. 0

d. 3

e. 6



$A_\Delta = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$

Green's Thm:

$\oint_C P\,dx + Q\,dy = \iint Q_x - P_y\,dA$

$P = \frac{y}{2} \quad Q = 0$

$\frac{dP}{dy} = \frac{1}{2} \quad \frac{dQ}{dx} = 0$

$\iint (0 - \frac{1}{2})\,dA = -\frac{1}{2}\iint dA = -\frac{1}{2}(6) = -3$

5. The surface S is parameterized by $\vec{r}(u,v) = \langle 2\sin(v)\cos(u), 2\sin(v)\sin(u), 2\cos(v)\rangle$, $0 \le u \le 2\pi$ and $0 \le v \le \pi$. Which of the following statements is/are correct?

✗ P. The surface S is the sphere ✓ centered at ✓ $(0,0,0)$ with radius $\cancel{4}$ 2.

✓ Q. The vector $\vec{r}_u(P)$ is parallel to the tangent plane to S at the point P.

✗ R. The area of the surface $S = \iint_D dA$, where $D = \{(u,v) \mid 0 \le u \le 2\pi,\ 0 \le v \le \pi\}$.

$\wedge$ $|\vec{r}_u \times \vec{r}_v|$

a. P only

b. Q only

c. R only

d. P and Q

e. Q and R

$\vec{n} = \vec{r}_u \times \vec{r}_v$ so by definition $\vec{r}_u, \vec{r}_v$ are parallel to a tangent plane

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6. Find the area of the surface S, where S is the part of the plane $2x + y + 2z = 10$ that lies inside the cylinder $x^2 + y^2 = 16$.

a. 48π

b. 18π

c. 16π

d. 12π

e. 24π

$$A = \iint_D |\vec{r}_u \times \vec{r}_v| \, dA$$

$$|\vec{r}_u \times \vec{r}_v| = |\vec{n}|$$

$$2z = 10 - 2x - y$$

$$z = 5 - x - \tfrac{1}{2}y$$

$$\vec{n} = \langle 1, \tfrac{1}{2}, 1 \rangle$$

$$|\vec{n}| = \sqrt{1 + \tfrac{1}{4} + 1} = \sqrt{\tfrac{9}{4}} = \tfrac{3}{2}$$

$$\iint \tfrac{3}{2}\, dx\,dy = \int_0^{2\pi}\int_0^4 \tfrac{3}{2} r\,dr\,d\theta = \int_0^{2\pi} \tfrac{3}{4} r^2 \Big|_0^4 d\theta$$

$$= \int_0^{2\pi} 12\, d\theta = 24\pi$$

7. If f is a potential function of $\vec{F}(x,y) = \langle -y\sin(xy), -x\sin(xy) - 2y\rangle$ and $f(0,0) = 3$, find $f(0,2)$.

$$\int -y\sin(xy)\,dx = \cos(xy) + C$$

$$\int -x\sin(xy) - 2y\,dy = \cos(xy) - y^2 + C$$

a. 1

b. 0

c. -1

d. -2

e. -3

$$f = \cos(xy) - y^2 + C$$

$$f(0,0) = 3 = \cos(0) - 0 + C$$

$$3 = 1 + C$$

$$2 = C$$

$$f = \cos(xy) - y^2 + 2$$

$$f(0,2) = \cos(0) - 4 + 2 = -1$$

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8. If $\vec{F} = \langle -x, 0, z\rangle$, which of the following must be correct?

P. The flux of $\vec{F}$ across the plane $z = 1$ is 0. $\vec{n} = \langle 0,0,1\rangle \cdot \langle -x,0,z\rangle = z \neq 0$

Q. The flux of $\vec{F}$ across the plane $x = 1$ is 0. $\vec{n} = \langle 1,0,0\rangle \cdot \langle -x,0,z\rangle = -x \neq 0$

R. The flux of $\vec{F}$ across a unit sphere is 0. $\operatorname{div} F = \nabla \cdot F = -1 + 0 + 1 = 0$

a. P only

b. Q only

c. R only

d. P and Q only

e. P, Q, and R

$$\text{Flux} = \oint F \cdot \vec{n}\,ds = \iint \operatorname{div} F\,dA$$

9. Let $\vec{F}(x,y,z) = \langle x, y, -2xy\rangle$. Evaluate the line integral $\int_C \vec{F}\cdot d\vec{r}$, where C is the curve parameterized by $\vec{r}(t) = \langle \cos t, \sin t, 2t\rangle$, $0 \le t \le \frac{\pi}{2}$.

a. $-\pi$

b. π

c. 2

d. -2

e. 0

$dr = \langle -\sin t, \cos t, 2\rangle$

$F = \langle \cos t, \sin t, -2\cos t\sin t\rangle$

$F\cdot dr = -\cos t\sin t + \cos t\sin t - 4\cos t\sin t$

$\int_0^{\pi/2} -4\cos t\sin t\,dt = \int_0^{\pi/2} -4\cos t\, u\cdot\frac{du}{\cos t} = \int_0^{\pi/2} -4u\,du$

$u = \sin t$
$du = \cos t\,dt$
$\frac{du}{\cos t} = dt$

$= -2u^2\Big|_0^{\pi/2} = -2\sin^2 t\Big|_0^{\pi/2} = -2(1-0) = -2$

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10. If the surface S is parameterized by $\vec{r}(u,v) = \langle u, v\cos(2u), v\sin(2u)\rangle$, find an equation of the tangent plane to S at the point $(\pi, 1, 0)$. $\rightarrow$ $u = x = \pi$

$y = 1 = v\cos(2\pi)$
$1 = v$ $\rightarrow (\pi, 1)$

a. $-2x + z + \pi = 0$

b. $-2x + z + 2\pi = 0$

c. $2x + z - 2\pi = 0$

d. $2y + z - \pi = 0$

e. $-2y + z + 2\pi = 0$

$\vec{n} = \vec{r}_u \times r_v$

$r_u = \langle 1, -2v\sin(2u), 2v\cos(2u)\rangle$

$r_v = \langle 0, \cos(2u), \sin(2u)\rangle$

$r_u \times r_v = \begin{vmatrix} i & j & k \\ 1 & -2v\sin(2u) & 2v\cos(2u) \\ 0 & \cos(2u) & \sin(2u)\end{vmatrix} = i(-2v\sin^2(2u) - 2v\cos^2(2u)) - j(\sin(2u)) + k(\cos(2u))$

$= \langle -2v, -\sin(2u), \cos(2u)\rangle$

$@ (\pi, 1) = \langle -2, 0, 1\rangle = \vec{n}$

$\langle -2, 0, 1\rangle \cdot \langle x-\pi, y-1, z-0\rangle = -2(x-\pi) + z - 0 \Rightarrow$

$-2x + 2\pi + z = 0$

11. Which of the following vector fields has the graph below?

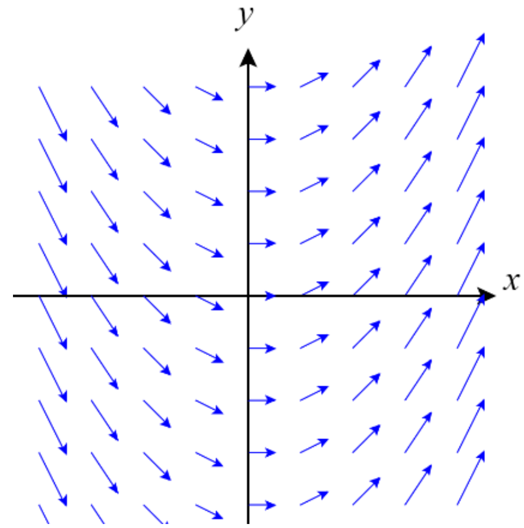
a. $\vec{F}_1(x,y) = -y\,\hat{\imath} + x\hat{\jmath}$

b. $\vec{F}_2(x,y) = \hat{\imath} + \hat{\jmath}$

c. $\vec{F}_3(x,y) = \hat{\imath} + x\,\hat{\jmath}$

d. $\vec{F}_4(x,y) = x\,\hat{\imath} + y\,\hat{\jmath}$

e. $\vec{F}_5(x,y) = y\,\hat{\imath} + \hat{\jmath}$



parallel in y-direction ⇒ independence of y

plug in points!

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12. Let $\vec{F}(x,y,z) = \langle 2y^3, 1, e^z\rangle$. Find the circulation of $\vec{F}$ along C, where C is the curve of intersection of the plane $y + 2z = 3$ and the cylinder $x^2 + y^2 = 4$. (Orient C to be counterclockwise when viewed from above.) By Stoke's Theorem,

$$\int_C \vec{F}\cdot d\vec{r} = \int \text{curl}\,F \cdot \vec{n}\,dS = \iint |\text{curl}\,F|\,dA$$

a. $\displaystyle\int_0^{2\pi}\int_0^2 -6r^3\sin^2\theta\,dr\,d\theta$

b. $\displaystyle\int_0^{2\pi}\int_0^2 -12r^3\sin^2\theta\,dr\,d\theta$

c. $\displaystyle\int_0^{2\pi}\int_0^2 12r^2\cos^2\theta\,dr\,d\theta$

d. $\displaystyle\int_0^{2\pi}\int_0^2 12r^3\cos^2\theta\,dr\,d\theta$

e. $\displaystyle\int_0^{2\pi}\int_0^2 -6r^2\sin^2\theta\,dr\,d\theta$

$$\text{curl}\,F = \begin{vmatrix} i & j & k \\ dx & dy & dz \\ 2y^3 & 1 & e^z \end{vmatrix} = i(0-0) - j(0-0) + k(0-6y^2)$$

$$= \langle 0, 0, -6y^2\rangle$$

$$\vec{n}\,dS = |\langle 0,0,-6y^2\rangle| = \sqrt{(-6y^2)^2} = -6y^2$$

$$\iint -6y^2\,dA = \int_0^{2\pi}\int_0^2 -6r^2\sin^2\theta\,r\,dr\,d\theta$$

13. Which of the following is correct?

a. If $\vec{F}$ is conservative, then $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$ for any two smooth curves C_1 and C_2.

only if C_1, C_2 have the same endpoints

b. If $\vec{F}$ is conservative, then $\iint_S \vec{F} \cdot d\vec{S} = 0$.

$= \iiint \operatorname{div} F \cdot dV$ and $\operatorname{div} F \neq 0$

c. If $\vec{F}$ is conservative, then $\operatorname{div} \vec{F} = 0$.

not necessarily

d. If D is a simply connected planar region, then the area of D is $\oint_{\partial D} \frac{y}{2} dx - \frac{x}{2} dy$, where ∂D is oriented counterclockwise.

e. If $\vec{F} = \langle x, -2y, z \rangle$, then $\oiint_S \vec{F} \cdot d\vec{S} = 0$, where S is a unit sphere.

$= \iiint \operatorname{div} F \cdot dV$

$\operatorname{div} F = 1 - 2 + 1 = 0$

* F conservative $\Rightarrow$ curl F = 0

$f = z + 1$

14. Set up a double integral for the surface integral $\iint_S (z+1)\, dS$, where S is the part of the paraboloid $z = x^2 + y^2 - 1$, $-1 \leq z \leq 5$.

$= \int f(r(t)) \cdot |\vec{r}_x \times \vec{r}_y| \, dA$

explicit surface

a. $\int_0^{2\pi} \int_0^1 r^3 \sqrt{4r^2+1}\, dr\, d\theta$

b. $\int_0^{2\pi} \int_0^1 r^2\, dr\, d\theta$

c. $\int_0^{2\pi} \int_0^{\sqrt{6}} r^2\, dr\, d\theta$

d. $\int_0^{2\pi} \int_0^{\sqrt{6}} r^3 \sqrt{4r^2+1}\, dr\, d\theta$

e. $\int_0^{2\pi} \int_0^{\sqrt{6}} r^2 \sqrt{4r^2+1}\, dr\, d\theta$

$\vec{r}_x \times \vec{r}_y = \langle -2x, -2y, 1 \rangle$

$|\vec{r}_x \times \vec{r}_y| = \sqrt{4x^2 + 4y^2 + 1}$

$= \sqrt{4r^2+1}$

$\iint [(x^2+y^2-1)+1] \cdot \sqrt{4x^2+4y^2+1}\, dA$

$= \iint r^2 \sqrt{4r^2+1}\, dA$

$= \int_0^{2\pi} \int_0^{\sqrt{6}} r^3 \sqrt{4r^2+1}\, dr\, d\theta$

$-1 = r^2 - 1$

$0 = r$

$5 = r^2 - 1$

$6 = r^2$

$\sqrt{6} = r$

15. Find the work done by the force $\vec{F} = \langle 3x^2, 3y^2 \rangle$ in moving a particle along the parametric curve $\vec{r}(t) = \langle t\cos t, t\sin t \rangle$, $0 \le t \le 2\pi$.

Hint: Is $\vec{F}$ conservative? yes $\Rightarrow$ use $\int_a^b \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

↑ potential function

a. $8\pi^3$

b. 8π

c. 2π

d. $2\pi^3$

e. 0

$$\left.\begin{array}{l}\int 3x^2\,dx = x^3 + C \\ \int 3y^2\,dy = y^3 + C\end{array}\right\} f = x^3 + y^3 + C$$

$$= t^3\cos^3 t + t^3\sin^3 t + C$$

$$f(b) - f(a) = (2\pi\cos(2\pi))^3 + (2\pi\sin(2\pi))^3 - 0$$

$$= 8\pi^3$$

---

16. Calculate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$ $= \oint_{\partial S} F \cdot dr$, where $\vec{F} = \langle y, -x, e^{xyz} \rangle$ and S is the part of paraboloid $z = 3x^2 + 3y^2$, $0 \le z \le 6$, oriented downward.

a. -2π

b. -4π

c. 0

d. 2π

e. 4π

$\partial S: \; 6 = 3r^2$

$\vec{r} = \langle \sqrt{2}\cos t, \sqrt{2}\sin t, 6 \rangle$ ↑ orientation

$\vec{r} = \langle -\sqrt{2}\cos t, -\sqrt{2}\sin t, -6 \rangle$ ↓ orientation

$dr = \langle \sqrt{2}\sin t, -\sqrt{2}\cos t, 0 \rangle$

$F = \langle \sqrt{2}\sin t, -\sqrt{2}\cos t, e^{2\cos t \sin t \cdot 6} \rangle$

$F \cdot dr = 2\sin^2 t + 2\cos^2 t + 0 = 2$

$\int_0^{2\pi} 2\,dt = 2t\Big|_0^{2\pi} = 4\pi$

17. Let $\vec{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3 \rangle$ and let S be the sphere centered at $(0, 0, 1)$ with radius 1. Then the flux of $\vec{F}$ across the surface S is

a. $\int_0^{2\pi} \int_0^{\pi} \int_0^{1} 3\rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

b. $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{\cos\phi} 3\rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

c. $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{\cos\phi} 3\rho^2 \, d\rho \, d\phi \, d\theta$

d. $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{2\cos\phi} 3\rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

e. $\int_0^{2\pi} \int_0^{\pi} \int_0^{2\cos\phi} 3\rho^2 \, d\rho \, d\phi \, d\theta$

flux $= \iiint \operatorname{div} F \cdot dV$

$\operatorname{div} F = 3x^2 + 3y^2 + 3z^2$

$= 3\rho^2$

$x^2 + y^2 + (z-1)^2 = \rho = 1$

$x^2 + y^2 + z^2 - 2z + 1 = 1$

$\rho^2 - 2\rho\cos\phi = 0$

$\rho - 2\cos\phi = 0$

$\rho = 2\cos\phi$

$* \; J = \rho^2 \sin\phi$

18. Which of the following regions is/are simply connected?

$D_1 = \{ (x, y) \mid 1 < x^2 + y^2 < 9 \text{ and } x > 0 \}$

$D_2 = \{ (x, y) \mid x > 0 \text{ and } y > 0 \}$

$D_3 = \{ (x, y) \mid y \neq 1 \}$

a. D_1 only

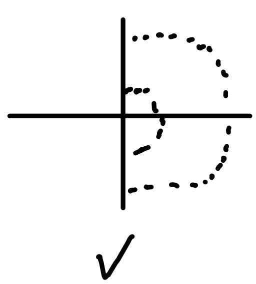
b. D_2 only

c. D_3 only

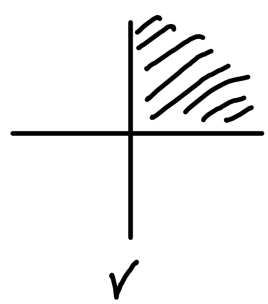
d. D_1 and D_2

e. D_2 and D_3

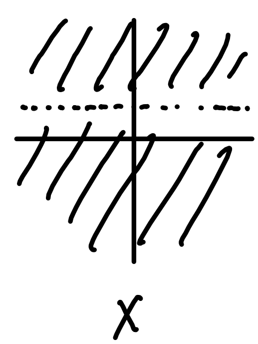
D_1:



D_2:



D_3:



19. Let D be the region bounded by $y = \sqrt{2x - x^2}$ and the x-axis and let ∂D be its boundary curve oriented positively. Then

$$\int_{\partial D} \underbrace{-x^2 y}_{P}\, dx + \underbrace{xy^2}_{Q}\, dy = \int \frac{dQ}{dx} - \frac{dP}{dy} = \int y^2 + x^2 = \int r^2\, r\, dr\, d\theta$$

a. $\displaystyle\int_0^{\pi} \int_0^{2\cos\theta} -r^2 \, dr \, d\theta$

b. $\displaystyle\int_0^{\pi/2} \int_0^{2\cos\theta} r^3 \, dr \, d\theta$

c. $\displaystyle\int_0^{\pi/2} \int_0^{2\cos\theta} -r^3 \, dr \, d\theta$

d. $\displaystyle\int_0^{\pi} \int_0^{2\cos\theta} r^3 \, dr \, d\theta$

e. $\displaystyle\int_0^{\pi/2} \int_0^{2\cos\theta} r^2 \, dr \, d\theta$

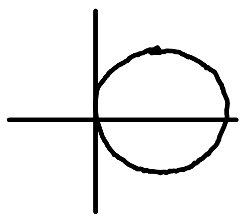
$y = \sqrt{2x - x^2}$

$y^2 = 2x - x^2$

$x^2 - 2x + 1 + y^2 = 1 \Rightarrow r^2 - 2r\cos\theta = 0$

$(x-1)^2 + y^2 = 1$ $\quad r - 2\cos\theta = 0$

$r = 2\cos\theta$



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20. Let $\vec{F}(x, y, z) = (3x - y)\,\hat{\imath} + (3y - z)\,\hat{\jmath} + (3z - x)\,\hat{k}$ and let S be the surface of the solid bounded by $x = 0,\ x = 2,\ y = 0,\ y = 2,\ z = 0$, and $z = 2$. Find the flux of $\vec{F}$ across S.

a. 54

b. 108

c. 36

d. 72

e. 18

flux $= \iiint \operatorname{div} F \cdot dA$

$\operatorname{div} F = 3 + 3 + 3 = 9$

$\int_0^2 \int_0^2 \int_0^2 9 \, dx\, dy\, dz$

$= \int_0^2 \int_0^2 18 \, dy\, dz = \int_0^2 36 \, dz = 72$

21. Let $\vec{F}(x,y,z) = \left\langle \cos\left(\sqrt{x^2+y^2+z^2}\right), e^{\sqrt{xyz}}, \sin^3(x) \right\rangle$. Find $\operatorname{div}(\operatorname{curl}\vec{F})$ at the point $(1,1,1)$.

a. 0

b. e

c. $\dfrac{e}{2}$

d. $\sqrt{e}$

e. 1

div(curl F) = 0 always.

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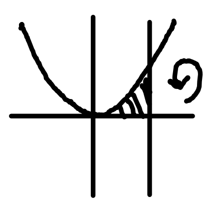
22. Let $\vec{F}(x,y) = \langle x^3 - y^2, x^2 - y^3 \rangle$. Let D be the region bounded by $y = x^2$, $y = 0$ and $x = 1$, and ∂D be its boundary curve oriented positively. Then the flux of $\vec{F}$ across the boundary curve $\partial D = \oint_{\partial D} \vec{F} \cdot \hat{n}\, ds =$



a. $\displaystyle\int_0^1 \int_0^{x^2} (2x+2y)\, dy\, dx$

b. $\displaystyle\int_0^1 \int_{x^2}^{1} (2x+2y)\, dy\, dx$

c. $\displaystyle\int_0^1 \int_0^{x^2} (3x^2+3y^2)\, dy\, dx$

d. $\displaystyle\int_0^1 \int_{x^2}^{1} (3x^2-3y^2)\, dy\, dx$

e. $\displaystyle\int_0^1 \int_0^{x^2} (3x^2-3y^2)\, dy\, dx$

flux = $\iint \operatorname{div} F \cdot dA$

$\operatorname{div} F = 3x^2 - 3y^2$

$\int_0^1 \int_0^{x^2} 3x^2 - 3y^2 \, dy\, dx$