Answer the following problems.

1. Calculate $\iint_S (z+1)dS$, where S is the part of the paraboloid $z = x^2 + y^2 - 1$, $-1 \leq z \leq 1$.

(a) (4 points) Use the cylindrical coordinates to find the parameterization of S

$x = r\cos\theta$

$y = r\sin\theta$

$z = x^2 + y^2 - 1 = r^2 - 1$

$-1 \leq z \leq 1$

$-1 \leq r^2 - 1 \leq 1$

$0 \leq r^2 \leq 2$

$0 \leq r \leq \sqrt{2}$ $\quad 0 \leq \theta \leq 2\pi$

$\vec{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, r^2-1 \rangle$

(b) (6 points) Set up a double integral for the surface integral

$\vec{r}_r = \langle \cos\theta, \sin\theta, 2r \rangle \qquad \vec{r}_\theta = \langle -r\sin\theta, r\cos\theta, 0 \rangle$

$$\vec{r}_r \times \vec{r}_\theta = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & 2r \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = -2r^2\cos\theta\, i + 2r^2\sin\theta\, j + (r\cos^2\theta + r\sin^2\theta)k$$

$= \langle -2r^2\cos\theta, 2r^2\sin\theta, r \rangle$

$$dS = |\vec{r}_r \times \vec{r}_\theta|\, dr\, d\theta = \sqrt{4r^4\cos^2\theta + 4r^4\sin^2\theta + r^2}\, dr\, d\theta$$

$$= \sqrt{r^2(4r^2+1)}\, dr\, d\theta = r\sqrt{4r^2+1}\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{\sqrt{2}} r^2\left(r\sqrt{4r^2+1}\right) dr\, d\theta = \int_0^{2\pi}\int_0^{\sqrt{2}} r^3\sqrt{4r^2+1}\, dr\, d\theta$$