Directions: Write your name and discussion time above. Solve the following problems. Show all your work, and clearly indicate your final answers.

1. Let $\vec{u} = \langle 1, -3, 2\rangle$ and $\vec{w} = \langle 2, -2, 1\rangle$.

(a) Find the vector of magnitude 2 in the opposite direction of $\vec{u} - 3\vec{w}$.

(b) Find vectors $\vec{F}_1$ and $\vec{F}_2$ such that $\vec{u} = \vec{F}_1 + \vec{F}_2$, $\vec{F}_1$ is parallel to $\vec{w}$, and $\vec{F}_2 \cdot \vec{w} = 0$.

[Note: Your answer for $\vec{F}_2$ does not need to be simplified.]

(a) $\vec{u} - 3\vec{w} = \langle 1, -3, 2\rangle - 3\langle 2, -2, 1\rangle = \langle 1, -3, 2\rangle - \langle 6, -6, 3\rangle$

$= \langle -5, 3, -1\rangle$

$|\langle -5, 3, -1\rangle| = \sqrt{(-5)^2 + (3)^2 + (-1)^2} = \sqrt{25+9+1} = \sqrt{35}$

$$\frac{-2(\vec{u} - 3\vec{w})}{|(\vec{u} - 3\vec{w})|} = \frac{-2\langle -5, 3, -1\rangle}{\sqrt{35}} = \frac{1}{\sqrt{35}}\langle 10, -6, 2\rangle$$

$$= \left\langle \frac{10}{\sqrt{35}}, \frac{-6}{\sqrt{35}}, \frac{2}{\sqrt{35}} \right\rangle$$

(b) $\vec{u} = \langle 1, -3, 2\rangle = \vec{F}_1 + \vec{F}_2$

$\vec{F}_1$ parallel to $\vec{w}$ $\Rightarrow$ use $\text{proj.}_{\vec{w}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{w}}{|\vec{w}|}\right)\left(\frac{\vec{w}}{|\vec{w}|}\right)$

$\vec{u}\cdot\vec{w} = \langle 1, -3, 2\rangle \cdot \langle 2, -2, 1\rangle = 1(2) + (-3)(-2) + 2(1) = 10$

$|\vec{w}| = \sqrt{(2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$

$F_1 = \text{proj.}_{\vec{w}}\vec{u} = \left(\frac{10}{3}\right)\left(\frac{1}{3}\right)\langle 2, -2, 1\rangle = \frac{10}{9}\langle 2, -2, 1\rangle \Rightarrow$

$F_1 = \left\langle \frac{20}{9}, \frac{-20}{9}, \frac{10}{9} \right\rangle$

$F_2 = \vec{u} - \vec{F}_1 = \langle 1, -3, 2\rangle - \left\langle \frac{20}{9}, \frac{-20}{9}, \frac{10}{9} \right\rangle = \left\langle \frac{-11}{9}, \frac{-7}{9}, \frac{8}{9} \right\rangle$