Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Simplify all solutions completely and clearly indicate your answers.

1. Let L be the line passing through the point $P_0(1, 1, 2)$ and perpendicular to the plane S_1 : $2x - y + 2z = 3$, and S_2 be the plane containing L and the point $Q_0(4, 1, 4)$.

(a) (3 points) Find the <u>vector equation</u> of the line L.

$S_1 = \langle 2, -1, 2 \rangle = \vec{n}_1 =$ direction vector $\rightarrow$ this is the normal vector to S_1, and since L is also perpendicular to S_1, $\vec{n}$ and L will be parallel, allowing us to use $\vec{n}$ as a direction vector for L.

$P_0 = (1, 1, 2)$

$\vec{V_0} = \langle 1, 1, 2 \rangle =$ position vector

$\vec{r} = \vec{V_0} + t\vec{n}_1$ where $\vec{V_0}$ is position vector and $\vec{n}_1$ is direction vector. $\}$ formula

$$\boxed{\vec{r} = \langle 1,1,2 \rangle + t\langle 2,-1,2 \rangle = \langle 1+2t, 1-t, 2+2t \rangle}$$

(b) (5 points) Find the <u>linear equation</u> of the plane S_2.

S_2 contains L and $(4,1,4)$

$$L = \begin{cases} x = 1+2t \\ y = 1-t \\ z = 2+2t \end{cases} \Rightarrow \langle 2,-1,2 \rangle = \vec{v} = \text{direction vector from (a)}$$

given ... L at $t=0$ (plug $t=0$ into parametric equation of L)

two points on S_2: $(4,1,4)$, $(1,1,2)$; make vector by subtracting:

$\vec{u} = \langle 1-4, 1-1, 2-4 \rangle = \langle -3, 0, -2 \rangle$

$$\vec{u} \times \vec{v} = \det\left(\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 0 & -2 \\ 2 & -1 & 2 \end{vmatrix} \right) = \hat{i}(0-2) - \hat{j}(-6+4) + \hat{k}(3-0)$$

$= -2\hat{i} + 2\hat{j} + 3\hat{k} = \langle -2, 2, 3 \rangle = \vec{n_2} =$ normal to S_2

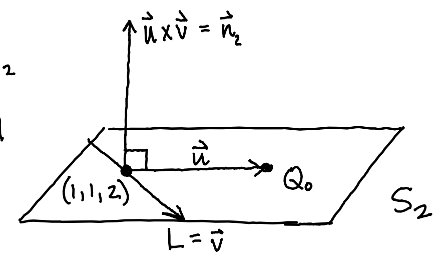
$-2x + 2y + 3z = d \leftarrow$ use point $(4,1,4)$ to find d

$-2(4) + 2(1) + 3(4) = d$

$-8 + 2 + 12 = 6 = d$

$$\boxed{-2x + 2y + 3z = 6}$$

$\uparrow$ equation of S_2



(c) (2 points) Show that S_1 is orthogonal to S_2.

$S_1 = \langle 2, -1, 2 \rangle$ $\qquad$ $S_2 = \langle -2, 2, 3 \rangle$

$S_1 \cdot S_2 = \langle 2,-1,2 \rangle \cdot \langle -2,2,3 \rangle = -4 + -2 + 6 = 0 \Rightarrow$

S_1 and S_2 are orthogonal ✓