Name: SOLUTIONS

Section:

MAC 2313
Discussion Quiz 3

$$\vec{r}(t) =< 2t^4 - 3, \ln(t) + 5, \frac{3}{2}t^2 + \frac{1}{2} >$$

1. $(5pts)$ Find parametric equations for the tangent line to curve $\vec{r}(t)$ at the point $(-1, 5, 2)$.

need tangent vector, <u>not</u> unit tangent vector

$T(t) = \vec{r}\,'(t) = \langle 8t^3, \frac{1}{t}, 3t \rangle$ ← do not divide by $|\vec{r}\,'(t)|$

point: $(-1, 5, 2)$ ← position
↳ this is at $t=1$ ($\ln t + 5 = 5 \Rightarrow \ln t = 0 \Rightarrow t = 1$)

$T(1) = \langle 8, 1, 3 \rangle$ ← direction

$$\begin{cases} x = -1 + 8t \\ y = 5 + t \\ z = 2 + 3t \end{cases}$$ ← parametric eauation of $T(1)$

↑ from point ↑ from $T(1)$

$$\vec{v}(t) =< \frac{3}{4}t^4, \frac{\sqrt{6}}{3}t^3, \frac{1}{2}t^2 >$$

2. $(5pts)$ Find the arc length of $\vec{v}(t)$ for $0 \le t \le 1$.

$f'(t) = 3t^3$ $\quad g'(t) = \sqrt{6}t^2$ $\quad h'(t) = t$

$$L = \int_0^1 \sqrt{(3t^3)^2 + (\sqrt{6}t^2)^2 + (t)^2}\, dt$$

$$= \int_0^1 \sqrt{9t^6 + 6t^4 + t^2}\, dt = \int_0^1 \sqrt{t^2(9t^4 + 6t^2 + 1)}\, dt$$

$$= \int_0^1 \sqrt{t^2(3t^2+1)^2}\, dt = \int_0^1 t(3t^2+1)\, dt = \int_0^1 3t^3 + t\, dt$$

$$= \frac{3}{4}t^4 + \frac{1}{2}t^2 \Big|_0^1 = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$$