

Key

Quiz 3C
MAC2313 L10

1. Find the domain and range of the following functions. You may write the domain as an inequality. Clearly label your answers.

(a) [2 pts] $f(x, y) = \sqrt{9 - 3x^2 - 3y^2}$

$$9 - 3x^2 - 3y^2 \geq 0$$

$$9 \geq 3x^2 + 3y^2$$

↓

$$\text{domain: } 3 \geq x^2 + y^2 \geq 0$$

$$\text{when } x^2 + y^2 = 0, f = \sqrt{9} = 3.$$

$$\text{when } x^2 + y^2 = 3, f = \sqrt{9-9} = 0$$

↓

$$\text{range: } [0, 3]$$

(b) [3 pts] $f(x, y, z) = \ln(25 - 5x^2 - y^2 - 5z^2)$

$$25 - 5x^2 - y^2 - 5z^2 > 0$$

$$\text{domain: } 25 > 5x^2 + y^2 + 5z^2 \geq 0$$

$$\text{when } 5x^2 + y^2 + 5z^2 = 0, f = \ln(25)$$

$$\text{as } 5x^2 + y^2 + 5z^2 \rightarrow 25, \text{ we have } \lim_{x \rightarrow 0} \ln(x) = -\infty$$

} so range
is $(-\infty, \ln(25)]$

2. [5 pts] Find the level curves of the function below by letting $k = 1, 2, 5$ in three separate equations. **Be sure to simplify.**

$$F(x, y) = \frac{20}{1 + 2x^2 + 5y^2} = k$$

$$1 = \frac{20}{1 + 2x^2 + 5y^2}$$

$$1 + 2x^2 + 5y^2 = 20$$

$$2x^2 + 5y^2 = 19$$

$$2 = \frac{20}{1 + 2x^2 + 5y^2}$$

$$2(1 + 2x^2 + 5y^2) = 20$$

$$1 + 2x^2 + 5y^2 = 10$$

$$2x^2 + 5y^2 = 9$$

$$5 = \frac{20}{1 + 2x^2 + 5y^2}$$

$$5(1 + 2x^2 + 5y^2) = 20$$

$$1 + 2x^2 + 5y^2 = 4$$

$$2x^2 + 5y^2 = 3$$

Since $ax^2 + by^2 = c^2$ is the standard equation for an ellipse

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The level curves form a family of which? **CIRCLE ONE:**

Hyperbolas

Parabolas

Ellipses