MAC 2313 Quiz 4 Version C

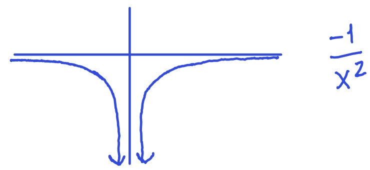
Name: Key

1. Find the following limit if it exists (otherwise, show that it does not exist).

$$\lim_{(x,y)\to(0,0)} e^{-\frac{1}{x^2+y^2}}$$

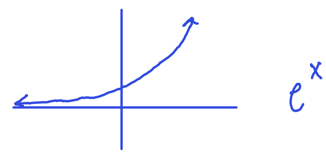
change to polar: $\lim\limits_{r\to 0} e^{\frac{-1}{r^2}}$

note the graph of $\frac{-1}{x^2}$:



so as $x \to 0$, $\frac{-1}{x^2} \to -\infty$

and $\lim\limits_{x\to -\infty} e^x = 0$



so we have $\lim\limits_{r\to 0} e^{\frac{-1}{r^2}} = 0$

* there are slightly different ways to do this problem but you will get the same answer.

2. For the function $f(x,y) = 3x^5y - 10x^3y^3 + 3xy^5$, calculate

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

$$\frac{\partial f}{\partial x} = 15x^4y - 30x^2y^3 + 3y^5$$

$$\frac{\partial^2 f}{\partial x^2} = 60x^3y - 60xy^3$$

$$\frac{\partial f}{\partial y} = 3x^5 - 30x^3y^2 + 15xy^4$$

$$\frac{\partial^2 f}{\partial y^2} = -60x^3y + 60xy^3$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 60x^3y - 60xy^3 - 60x^3y + 60xy^3 = 0$$