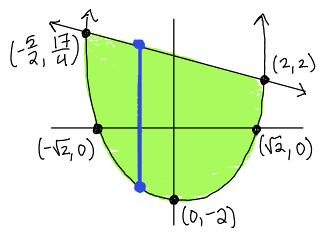
MAC2313 Quiz 5A

Name and section: Key

1). (7 points) Calculate the integral of the function $f(x,y) = x + y$ inside the region bounded by the parabola $y = x^2 - 2$ and the line $y = -(1/2)x + 3$.

$$\int_{-5/2}^{2}\int_{x^2-2}^{-x/2+3} (x+y)\,dy\,dx$$



For bounds:

$$x^2 - 2 = \frac{-x}{2} + 3$$
$$x^2 + \frac{x}{2} - 5 = 0$$
$$2x^2 + x - 10 = 0$$
$$(2x+5)(x-2) = 0$$
$$x = \frac{-5}{2} \qquad x = 2$$

$$\int_{-5/2}^{2}\left[xy + \frac{1}{2}y^2\right]\Big|_{x^2-2}^{-x/2+3} dx$$

$$\int_{-5/2}^{2} x\left(\frac{-x}{2}+3\right) + \frac{1}{2}\left(\frac{-x}{2}+3\right)^2 - \left(x(x^2-2) + \frac{1}{2}(x^2-2)^2\right) dx$$

$$\int_{-5/2}^{2} \frac{-x^2}{2} + 3x + \frac{1}{2}\left(\frac{x^2}{4} - 3x + 9\right) - x^3 + 2x - \frac{1}{2}\left(x^4 - 4x^2 + 4\right) dx$$

$$\int_{-5/2}^{2} \frac{-1}{2}x^4 - x^3 + x^2\left(\frac{-1}{2} + \frac{1}{8} + 2\right) + x\left(3 - \frac{3}{2} + 2\right) + \frac{9}{2} - 2 \quad dx$$

$$\int_{-5/2}^{2} \frac{-1}{2}x^4 - x^3 + \frac{13}{8}x^2 + \frac{7}{2}x + \frac{5}{2} \quad dx$$

$$\left[\frac{-1}{10}x^5 - \frac{1}{4}x^4 + \frac{13}{24}x^3 + \frac{7}{4}x^2 + \frac{5}{2}x\right]\Big|_{-5/2}^{2} \longleftarrow$$

THIS QUIZ ONLY: last step needed for full credit

2). (3 points) Convert the following from spherical (ρ, θ, ϕ) to rectangular (x, y, z) coordinates:

$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi$$

$$\left(\underset{\rho}{5}, \underset{\theta}{\frac{\pi}{2}}, \underset{\phi}{\frac{\pi}{4}}\right)$$

$$x = 5\sin(\pi/4)\cos(\pi/2) = 0$$

$$y = 5\sin(\pi/4)\sin(\pi/2) = 5\left(\sqrt{2}/2\right) = \frac{5\sqrt{2}}{2}$$

$$z = 5\cos(\pi/4) = \frac{5\sqrt{2}}{2}$$