MAC 2313 — **Quiz 5B**
February 20, 2024

KEY

1. (**5 points**) Calculate $\frac{dz}{dt}$, where

$$z = f(x, y) = x^2 - xy + y^2$$
$$x = e^t, y = e^{-t}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = 2x - y \qquad \frac{\partial z}{\partial y} = 2y - x$$

$$\frac{dz}{dt} = (2x-y)e^t - (2y-x)e^{-t}$$

$$\frac{dx}{dt} = e^t \qquad \frac{dy}{dt} = -e^{-t}$$

* need to have x, y in terms of t:

$$\frac{dz}{dt} = (2e^t - e^{-t})e^t - (2e^{-t} - e^t)e^{-t}$$

← this would get full credit on a quiz, but best to simplify whenever you can (especially on a test)

$$= 2e^{2t} - e^0 - 2e^{-2t} + e^0$$

$$= 2(e^{2t} - e^{-2t})$$

2. (**5 points**) Find the maximum rate of change of $f(x, y, z) = 3xyz$ at $(1, 0, 4)$ and the direction in which it occurs.

$$\nabla f = \langle f_x, f_y, f_z \rangle$$

$$\nabla f = \langle 3yz, 3xz, 3xy \rangle$$

$$\nabla f(1,0,4) = \langle 0, 12, 0 \rangle$$

$$|\nabla f(1,0,4)| = |\langle 0, 12, 0 \rangle| = \sqrt{0^2 + 12^2 + 0^2} = 12$$

$\therefore$ the max. rate of change is 12 in the direction $\langle 0, 12, 0 \rangle$

* If you gave the <u>unit</u> direction vector $\frac{1}{12}\langle 0, 12, 0 \rangle = \langle 0, 1, 0 \rangle$ that is also correct, just not necessary for this problem.