MAC2313 Key **Quiz 5** **27 February 2025**

Answer the following problems. Please show all of your work. Simplify all solutions completely and clearly indicate your answers

1. Let $f(x,y,z) = x^2 + y^3 + z^2$. Find the directional derivative of f in the direction of $< 0, 1, 1 >$ at the point $(-1, 1, 2)$.

$D_{\hat{u}} f(a,b,c) = \nabla f(a,b,c) \cdot \hat{u}$

$u = \langle 0, 1, 1 \rangle \quad |u| = \sqrt{1+1} = \sqrt{2} \quad \hat{u} = \langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\nabla f = \langle 2x, 3y^2, 2z \rangle \quad \nabla f(-1,1,2) = \langle -2, 3, 4 \rangle$

$D_{\hat{u}} f(-1,1,2) = \langle -2, 3, 4 \rangle \cdot \langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$= \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{7}{\sqrt{2}}$

$= \frac{7\sqrt{2}}{2}$ if you like, but $\frac{7}{\sqrt{2}}$ is fine

2. The point $(2, c, 2)$ is on the tangent plane of the surface $2xy + 3yz - zx = 0$ at $(1, 0, 1)$. Find c.

$\nabla f = \langle 2y - z, 2x + 3z, 3y - x \rangle$

$\nabla f(1,0,1) = \langle -1, 5, -1 \rangle$

$\langle -1, 5, -1 \rangle \cdot \langle x-1, y, z-1 \rangle = 0$

$-x + 1 + 5y - z + 1 = 0$

$-x + 5y - z = -2$

tangent plane

plug in $(2, c, 2)$ to plane:

$-2 + 5c - 2 = -2$

$5c = 2$

$c = \frac{2}{5}$