Answer the following problems. No calculators, formula sheets, or other aids are permitted. Please show all of your work. Each question is worth 5 points.

1. Let R be the region bounded by $x+2y=2$, $x-y=0$, $x+2y=5$, $x-y=1$. Use a transformation to evaluate the integral:

$$\iint_R 4(x-y)\,dA$$

$u = x+2y \qquad v = x-y$

$2 \le u \le 5 \qquad 0 \le v \le 1$

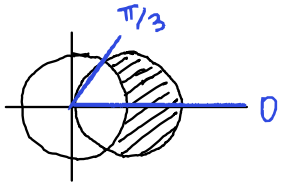
$$J(u,v) = \left(\det\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix}\right)^{-1} = \left(\det\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}\right)^{-1}$$

$$= (-1-2)^{-1} = (-3)^{-1} = \frac{-1}{3}$$

$$\int_2^5 \int_0^1 4v\left(\tfrac{1}{3}\right) dv\,du = \int_2^5 \int_0^1 \tfrac{4}{3} v\, dv\,du = \int_2^5 \tfrac{2}{3}v^2 \Big|_0^1 du =$$

$$\int_2^5 \tfrac{2}{3}\, du = \tfrac{2}{3}u \Big|_2^5 = \tfrac{10}{3} - \tfrac{4}{3} = \tfrac{6}{3} = 2$$

2. Set up, but do NOT evaluate a double integral to find the area inside the circle $(x-2)^2+y^2=4$ and outside the circle $x^2+y^2=4$ **in the first quadrant**.



$x^2+y^2=4$

$r^2=4$

$r=2$

$(x-2)^2+y^2=4$

$(r\cos\theta-2)^2+r^2\sin^2\theta=4$

$r^2\cos^2\theta-4r\cos\theta+4+r^2\sin^2\theta=4$

$r^2(\cos^2\theta+\sin^2\theta)-4r\cos\theta=0$

$r^2=4r\cos\theta$

$r=4\cos\theta$

$2=4\cos\theta$

$\frac{1}{2}=\cos\theta \Rightarrow \theta=\frac{\pi}{3}$ (first quadrant only)

$$\int_0^{\pi/3} \int_2^{4\cos\theta} r\,dr\,d\theta$$