Name: Key

Quiz 6B

MAC2313

1. [5 pts] Find and classify all critical points of $f(x, y) = 2x^4 + y^4$.

$f_x = 8x^3$ $\quad$ $f_y = 4y^3$ $\quad$ $f_{xx} = 24x^2$

$0 = 8x^3$ $\quad$ $0 = 4y^3$ $\quad$ $f_{yy} = 12y^2$

$0 = x$ $\quad$ $0 = y$ $\quad$ $f_{xy} = 0$

point $(0,0)$

Discriminant:

$D = f_{xx} f_{yy} - (f_{xy})^2$

$D(0,0) = 0 \cdot 0 - (0)^2 = 0 \Rightarrow$ test inconclusive

$f(x,y) = 2x^4 + y^4 \geq 0$ for all (x,y), so $(0,0)$ is an absolute minimum

2. [5 pts] Find the maximum and minimum values of $f(x, y, z) = x + y + z$ subject to $x^2 + y^2 + z^2 = 12$.

$f(x,y,z) = x + y + z$

$\nabla f = \langle 1, 1, 1 \rangle$

$g(x) = x^2 + y^2 + z^2 - 12$

$\nabla g = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g$

$\langle 1, 1, 1 \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$1 = \lambda 2x$ $\quad$ $1 = \lambda 2y$ $\quad$ $1 = \lambda 2z$

$\frac{1}{2\lambda} = x$ $\quad$ $\frac{1}{2\lambda} = y$ $\quad$ $\frac{1}{2\lambda} = z$

using $g(x)$:

$\left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = 12$

$3\left(\frac{1}{4\lambda^2}\right) = 12$

$\frac{1}{\lambda^2} = 16$

$\lambda^2 = \frac{1}{16}$

$\lambda = \pm \frac{1}{4}$

@ $\lambda = \frac{1}{4}$, $x = y = z = 2$

$f(2,2,2) = 6 \longleftarrow$ max. value of f

@ $\lambda = \frac{-1}{4}$, $x = y = z = -2$

$f(-2,-2,-2) = -6 \longleftarrow$ min value of f