Name: **Key** Quiz 6 MAC 2313

1. Set up a double integral (do not compute) in order to find the volume of the solid that is bounded by $z = x^2 + y^2$ and $z = 4 - x^2$. Show your work on how you got your bounds.

- plug in $(x,y) = (0,0)$ to see which is top function:

$$\left.\begin{array}{l} z = x^2 + y^2 \text{ gives } z = 0 \longrightarrow \text{lower} \\ z = 4 - x^2 \text{ gives } z = 4 \longrightarrow \text{upper} \end{array}\right\} \text{will use this for the integrand}$$

dxdy

- look at intersection, solve for x:

$$x^2 + y^2 = 4 - x^2$$
$$2x^2 = 4 - y^2$$
$$x^2 = 2 - \left(\frac{y^2}{2}\right)$$
$$x = \pm\sqrt{2 - (y^2/2)}$$

- Now let $x = 0$ to get y-bounds:

$2x^2 = 4 - y^2$ with $x = 0$ gives

$$y^2 = 4$$
$$y = \pm 2$$

$$\int_{-2}^{2} \int_{-\sqrt{2-(y^2/2)}}^{\sqrt{2-(y^2/2)}} \overset{4-2x^2-y^2}{\overset{\parallel}{\left[4 - x^2 - (x^2 + y^2)\right]}}\, dx\, dy$$

dydx

- look at intersection, solve for y:

$$x^2 + y^2 = 4 - x^2$$
$$y^2 = 4 - 2x^2$$
$$y = \pm\sqrt{4 - 2x^2}$$

- Now let $y = 0$ to get x-bounds:

$$0 = 4 - 2x^2$$
$$2x^2 = 4$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{4-2x^2}}^{\sqrt{4-2x^2}} \overset{4-2x^2-y^2}{\overset{\parallel}{\left[4 - x^2 - (x^2 + y^2)\right]}}\, dy\, dx$$

THERE IS A QUESTION ON THE BACK.

2.

(a) Identify the following surface given in cylindrical coordinates:

$$r^2 = z^2$$

$$x^2 + y^2 = z^2$$

cone

(b) Identify the following surface given in spherical coordinates:

$$\rho^2 \sin^2 \phi = 1$$

$$x^2 + y^2 = 1$$

cylinder

(c) Identify the following surface given in spherical coordinates:

$$\rho^2 = 2\rho \cos \phi$$

$$x^2 + y^2 + z^2 = 2z$$

$$x^2 + y^2 + z^2 - 2z = 0$$

$$x^2 + y^2 + z^2 - 2z + 1 = 1$$

$$x^2 + y^2 + (z-1)^2 = 1$$

sphere