Directions: Write your name above. Solve the following problems. Show all work, and clearly indicate final answers.

1. (5 pts) Find a potential function for the conservative vector field below:

$$\vec{F}(x,y,z) = \langle 3x^2 z,\ z^3 e^{6y},\ x^3 + \frac{1}{2} z^2 e^{6y} - 2z \rangle$$

Since $\vec{F}$ is conservative, we can find f.

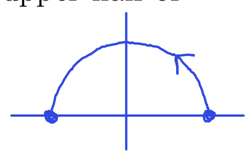
$\int P\,dx = \int 3x^2 z\,dx = x^3 z + C$

$\int Q\,dy = \int z^3 e^{6y}\,dy = \frac{1}{6} z^3 e^{6y} + C$

$\int R\,dz = \int x^3 + \frac{1}{2} z^2 e^{6y} - 2z\,dz = x^3 z + \frac{1}{6} z^3 e^{6y} - z^2 + C$

$f = x^3 z + \frac{1}{6} z^3 e^{6y} - z^2 + C$

2. (5 pts) Compute $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x,y) = \langle -xy,\ x^2 \rangle$ and C is the upper half of the circle $x^2 + y^2 = 4$ oriented counterclockwise.



Since we have a counterclockwise circle, we have parameters $x = 2\cos\theta$ and $y = 2\sin\theta$.

So $\vec{r}(\theta) = \langle 2\cos\theta,\ 2\sin\theta \rangle$ and $\vec{r}\,'(\theta) = \langle -2\sin\theta,\ 2\cos\theta \rangle$

$\vec{F}(\theta) = \langle -4\cos\theta\sin\theta,\ 4\cos^2\theta \rangle$

$\vec{F} \cdot \vec{r}\,' = 8\sin^2\theta\cos\theta + 8\cos^3\theta = 8\cos\theta(\sin^2\theta + \cos^2\theta) = 8\cos\theta$

$\int_0^{\pi} 8\cos\theta\,d\theta = 8\sin\theta \Big|_0^{\pi} = 8(0) - 8(0) = 0$