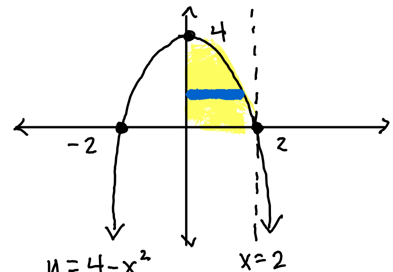
Name: Key

Quiz 7

1. (**5 points**) Evaluate the iterated integral $\int_0^2 \int_0^{4-x^2} xe^{x^2}\,dydx$ by reversing the order of integration.

from the original bounds we have

$y = 4 - x^2$, $y = 0$, $x = 2$, $x = 0$ $\Longrightarrow$



$y = 4 - x^2$

$x^2 = 4 - y$

$x = \sqrt{4-y}$

New integral:

$$\int_0^4 \int_0^{\sqrt{4-y}} xe^{x^2}\,dx\,dy$$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$= \int_0^4 \int_0^{\sqrt{4-y}} xe^u \frac{du}{2x}\,dy$$

$$= \int_0^4 \left[\tfrac{1}{2}e^u\right]_0^{\sqrt{4-y}} dy = \int_0^4 \left[\tfrac{1}{2}e^{x^2}\right]_0^{\sqrt{4-y}} dy$$

$$= \int_0^4 \tfrac{1}{2}e^{4-y} - \tfrac{1}{2}\,dy = \left[\tfrac{-1}{2}e^{4-y} - \tfrac{1}{2}y\right]_0^4$$

$$= \tfrac{-1}{2}e^0 - \tfrac{1}{2}(4) - \left[\tfrac{-1}{2}e^4 - 0\right] = \tfrac{-1}{2} - 2 + \tfrac{1}{2}e^4 = \tfrac{1}{2}e^4 - \tfrac{5}{2}$$

2. (**5 points**) Write in terms of inequalities involving spherical coordinates a description of the solid E that lies above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = 2z$.

$$\begin{bmatrix} x = \rho\cos\theta\sin\phi \\ y = \rho\sin\theta\sin\phi \\ z = \rho\cos\phi \\ x^2+y^2+z^2 = \rho^2 \end{bmatrix}$$

$z = \sqrt{x^2+y^2}$

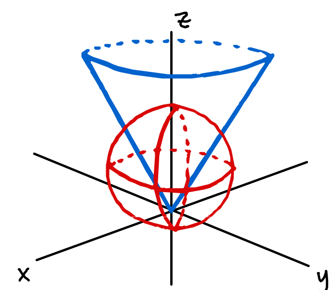
$\rho\cos\phi = \sqrt{\rho^2\cos^2\theta\sin^2\phi + \rho^2\sin^2\theta\sin^2\phi}$

$\rho\cos\phi = \sqrt{\rho^2\sin^2\phi\,(\cos^2\theta + \sin^2\theta)}$

$\rho\cos\phi = \rho\sin\phi$

$1 = \tan\phi$

$\phi = \frac{\pi}{4}, \frac{5\pi}{4}$



$x^2+y^2+z^2 = 2z$

$\rho^2 = 2[\rho\cos\phi]$

$\rho = 2\cos\phi$

$0 \le \rho \le 2\cos\phi$

$0 \le \phi \le \frac{\pi}{4}$

because ϕ must be less than π always

* E is bounded by a cone and sphere, both of which are circular. $z \ge 0$ by $z = \sqrt{x^2+y^2}$ but x, y can take any value. Thus

$0 \le \theta \le 2\pi$ to cover the entire region.