Name: Key

Quiz 8A

MAC2313 L29-L31

1. [6 pts] Use Green's Theorem to evaluate the given line integral along the positively oriented circle curve $C : x^2 + y^2 = 16$. **HINT:** Use polar coordinates after applying Green's Theorem.

$$\oint_C \frac{-y^3}{3}dx + \frac{x^3}{3}dy$$

$P = \frac{-y^3}{3}$ $\quad Q = \frac{x^3}{3}$

$\frac{dP}{dy} = -y^2$ $\quad \frac{dQ}{dx} = x^2$

$\iint \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA$

$x^2 + y^2 = 16$

$r = 4$

$\iint x^2 - (-y^2)\,dxdy = \iint x^2 + y^2\,dxdy = \int_0^{2\pi}\int_0^4 (r^2)r\,drd\theta =$

$\int_0^{2\pi}\int_0^4 r^3\,drd\theta = \int_0^{2\pi} \frac{r^4}{4}\Big|_0^4 d\theta = \int_0^{2\pi} 64\,d\theta =$

$64\theta \Big|_0^{2\pi} = 128\pi$

2. [4 pts] Use cylindrical coordinates to find a parametric representation for the part of the plane $z = x + 5$ that lies inside the cylinder $x^2 + y^2 = 64$. Include r, θ bounds.

$x = r\cos\theta$

$y = r\sin\theta$

$z = x + 5 = r\cos\theta + 5$

$x^2 + y^2 = 64$

$0 \le r \le 8$

$0 \le \theta \le 2\pi$

note that we do not let $r = 8$ here because we are looking at a disk, not just a circle